XXXVIII. Experimental Researches on the Strength of Pillars of Cast Iron from various Parts of the Kingdom. By Eaton Hodgkinson, Esq., F.R.S., F.G.S., Hon. Mem. Inst. Civ. E., Hon. Mem. Soc. Ing. Civ. Paris, and Professor of the Mechanical Principles of Engineering in University College, London.

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HAVING in the year 1840 offered to the Royal Society an extensive research upon this subject, which was honoured with the kindest notice of the Society, I felt grateful for the reception it had met with; and though in its preparation it had occupied my leisure time for some years, and contained the results of as many as 277 experiments, which I had made to prove the conclusions arrived at in it, I was still very anxious to improve Indeed the importance of the subject would seem to justify every effort and extend it. I could make for the purpose, when it is considered that a large portion of the houses, warehouses and shops in London, Manchester, Liverpool and throughout the country, depend for their principal supports upon iron pillars, which frequently appear very thin for the weight they have to bear, and being hollow do not allow us to judge from their appearance how small a quantity of metal they have in them, or in other words, whether the building is abundantly strong, or is ready to fall down and crush the persons within it, as has frequently happened to warehouses and other buildings dependent on iron Some of the pillars are made to pass through more than one story, or even are based on the foundation, and support an intermediate floor and the roof.

The importance of the subject, in a practical point of view at least, rendered it desirable that a number of pillars of large size should be broken, to obtain data for the application of the principles established in the preceding research; but this was impracticable at that time, notwithstanding the liberality of Mr. Fairbairn, who bore the expense of that inquiry. For by Mr. Fairbairn's lever then used, more than 18 tons could not be safely applied, and the iron box or frame in which the pillars were broken did not admit pillars of greater length than  $7\frac{1}{2}$  feet; but the laborious inquiry in which I was afterwards engaged by Mr. Stephenson, for investigating the properties of the Menai and Conway tubular bridges (that over the Conway in particular), required larger and more powerful apparatus than the preceding, and I can now apply more than three times the pressure formerly used, and break pillars of 10 feet long, and any shorter lengths, with even more accuracy than before.

A drawing of the former machine is given with the paper (Philosophical Transactions, 1840), and of the latter, with my experiments in the Report of the Commissioners appointed to inquire into the application of Iron to Railway Structures, 1849. The principle of both machines is the same; the present is a single lever, about 17 feet,

6 inches long, acting (usually) with a leverage of 8.4 upon the upper end of a strong bolt, which moves in a cylinder, just fitting it and keeping it always in the straight line of its pressure. The top of the bolt is rounded, so that the pressure may act through the axis, and the bottom is flat, and made horizontal, that it may press on the top of a vertical pillar, enclosed in a strong box, as before. See Plate XXXI.

The present experiments, and others in progress, were undertaken at the suggestion of Mr. Robert Stephenson, M.P., F.R.S., who knew that I had several experimental researches unfinished. He has generously contributed £200 towards the expense of the inquiry, and the Royal Society has added £200\*. By the kind permission of the Council of University College, London, my apparatus has been set up at the College, and the experiments made there.

I have in the present research broken only larger pillars than before, having had them cast 10 feet long, and from  $2\frac{1}{2}$  to 4 inches in diameter, solid and hollow. The first series includes twenty pillars (twelve of 10 feet long, and eight shorter ones cut out of them); the whole of these were supplied by the proprietors of the Low Moor Iron-works, for which no charge has yet been made  $\dagger$ . The iron of which these, with five exceptions, were made, was Low Moor iron, No. 2. The former research of 1840 was from Low Moor iron, No. 3; and the difference of the two is not very great, though the No. 2, as now used, is somewhat stronger than the other to bear a strain as a long pillar (p. 864), though softer than it and more easily crushed. The results of these experiments have been applied to the formulæ in my former research, which have undergone no change, except in the constants.

The former researches, and the formulæ to which they were reduced, had been applied to pillars of Low Moor iron only, with one exception; but though we could compute by them the strength for Low Moor iron, we had no evidence as to what were the relative strengths of pillars of the irons of the kingdom in general; and, at my request, Messrs. Easton and Amos have had cast for this purpose twenty-two solid pillars, each 10 feet long and  $2\frac{1}{2}$  inches in diameter, of eleven kinds of iron, making thirteen kinds with two I have had from Low Moor. The comparison of the breaking weights of these pillars shows that the strengths of the irons vary at least as 2 to 3, and their crushing strengths as 5 to 9 nearly.

In carrying out the experiments, in taking the dimensions, and applying the results to the formulæ in my research above mentioned, I have had the assistance of Mr. John Bridge, M.A., of University College, my health and eyesight not being sufficient to enable me to bear the labour alone.

- \* Since finishing the experiments in the present research, except those contained in the short Appendix, another £100 has been contributed by the Royal Society, and £100 more by Mr. Stephenson, with which I am making experiments on pillars of timber, &c.
- † A good balk of Memel timber was likewise sent gratuitously for experiment by the late Mr. Thomas Cubit, of Thames Bank, but his death prevented me having the power to thank him. His executors have liberally sent other timber for the purpose.
  - ‡ The pillars were cast 12 feet long, to increase their soundness, and afterwards cut to 10 feet long.

1. Before referring to the experiments which form the basis of the present memoir, I will give some notice of the principal matters contained in the former, but not sufficiently dilated upon; especially as they have an important bearing upon what will follow, and will themselves receive further illustration, or perhaps be capable of demonstration.

The profound and beautiful researches of Euler on the strength of pillars \* are entirely theoretical; they proceed on the supposition that the strength of a pillar is bounded by its power of resisting being bent out of a straight line, the resistance to incipient flexure being supposed to be the measure of its strength, which in long pillars, whose length is l and external and internal diameters d and  $d_1$ , is  $\frac{d^4}{l^2}$  or  $\frac{d^4-d_1^4}{l^2}$ , according as they are solid or hollow †.

- 2. In commencing experiments, in my former research, on this subject, and keeping in view the theory of Euler, I sought with great care for the weight which would produce incipient flexure in columns, and more particularly in those of cast iron. In this metal flexure commenced with very small weights, much smaller than would be useful to load pillars with in practice; and I became convinced that no such point existed in cast iron, or at any rate none that would be useful to the engineer; and my subsequent experiments upon wrought-iron pillars have been attended with very little more success, in seeking for the weight producing incipient flexure. Having been unsuccessful in seeking for the weight which would first produce flexure in columns, and being convinced that if it were found it would be of little or no use in practice, I sought in future for the weights necessary to break the pillars tried, and in most instances for the deflection and decrement of length produced by the weights laid on.
- 3. The pillars broken were placed in a vertical position during the experiments, and their ends were pressed between two horizontal plates of hardened steel, which, from the nature of the machine, were kept perfectly parallel. It was found that in cylindrical pillars of cast iron, whose ends were turned perfectly flat, parallel and perpendicular to the axis, the breaking weight varied as the 3.55th power of the diameter nearly, and inversely as the 1.7th power of the length, or as  $\frac{d^{3.55}}{l^{1.7}}$ , instead of  $\frac{d^4}{l^2}$ , as in the theory of Euler. In hollow pillars it is as  $\frac{d^{3.55}-d_1^{3.55}}{l^{1.7}}$ . The fractional indices above were found to apply to all solid pillars of cast iron, whose lengths varied from 120 times the diameter to as low as 30 times the diameter, or thereabouts. For pillars shorter than these it will be necessary to correct the results obtained, by considerations dependent on the crushing strength of the material. See pp. 403 to 406 of the former research...
- 4. The first experiments were made upon long uniform pillars, with their ends rounded in such a manner that the pressure applied to them would act in the direction of their axis, to meet the requirements of EULER's theory of the strength of pillars; but

<sup>\*</sup> Berlin Memoirs, 1757.

<sup>†</sup> Poisson, Mécanique, 2nd edition.

<sup>‡</sup> Philosophical Transactions, 1840, Part II.

it soon became evident that pillars of the same size with their ends flat, and well supported or bedded throughout, would require a much greater weight to break them than those with rounded ends; a large number of both kinds were made and broken by experiment to determine their relative strength to resist fracture, and it was found that so long as the pressure required had not been more than about one-fourth of that necessary to crush short specimens of the same material, the strength of a solid pillar with flat ends was a little more than three times that of one with rounded ends, the mean being 3·167, as shown in the abstract, page 387 of that research, but in other experiments it is still nearer to 3.

- 5. If the pressure necessary to break the pillar were much greater than about one-fourth of that which would crush short specimens of the same material, then the ratio 1 to 3 would become 1 to 2, or even less.
- 6. In the following abstract, pages 856 and 857, which includes experiments on hollow pillars with rounded and with flat ends, as above, and upon solid pillars of wrought iron and timber, the ratio of the strengths of the two kinds of pillars is nearly as 1 to 3, varying not widely from the former. From this abstract we see that pillars which have been loaded with more than about one-fourth of the crushing weight are in some degree injured, and the ratio in them differs from 1:3, becoming  $1:2\cdot6$ ,  $1:2\cdot4$ , &c.

In the last column of the abstract, page 857, denominated  $\frac{B}{C}$ , it may be seen at a glance what portion of the crushing weight was required to break each pillar; and in those where it amounted to 282, 272, or above 25\*, they were generally injured by the pressure.

- 7. In the abstract, p. 393 of the former research, the relative strengths of pillars—with both ends rounded, one end rounded and one end flat, and both ends flat,—are determined in a great number of experiments, and found to be nearly as 1, 2, 3, from experiments on cast iron, wrought iron, steel and timber.
- 8. In the abstract, page 391 of the former research, pillars  $60\frac{1}{2}$  inches long, with their ends turned flat, are compared, as to their strengths, with pillars of the same size with discs on their ends turned flat: the strengths of these two classes of pillars differed but little, but the pillars with discs on the ends are rather the stronger. In this abstract it was shown that pillars one-half the length of the preceding, and of the same diameter but with the ends rounded, were of the same strength as the preceding ones nearly, as shown by many experiments.

Attempt to account for the principal Results in the preceding description of the abstracts.

- 1. Of the general conclusion arrived at in the abstract last referred to, I would offer the following as an explanation. Suppose a long uniform bar of cast iron were bent by a pressure at its ends so as to take the form AbcdefB, where all the curves Abc, cde, efB, separated by the straight line AceB, would be equal, since the bar was supposed to
  - \* The pillars of the "Second London Mixture," 6 feet 3 inches long, gave the same result when  $\frac{B}{C}$ =-26.

be uniform. The curve having taken this form, suppose it to be rendered immoveable at the points b and f, by some firm fixings at those points. This done, it is evident we may remove the parts near to A and B, without at all altering the curve bedef of the part of the pillar between b and f, and consider only that part. The part bf, which alone we shall have to consider, will be equally bent at all the points b, d, f. The points c and e too are points of contrary flexure, consequently the pillar is not bent in them. These points are unconstrained except by the pressure which forces them together, and the pillar might be reduced to any degree in them, provided they were not crushed or detruded by the compressing force. These points may then be conceived as acting like the rounded ends of the pillars in Table I. in the former research, and the part cde of the pillar, with its ends c and e rounded, will be bearing the same weight as the whole pillar bcdef of double the length with its ends b, f firmly fixed.

- 2. That the strain at all the points b, d, f is nearly equal, is shown by subsequent experiments, in Tables V. and VI., upon pillars varying in length from 6 feet 3 inches to 5 feet and 2 feet 6 inches; that fracture in some cases took place at all three points at the same time, and in others at two of them, showing the fracture to have commenced on opposite sides of the middle and of the ends, as might be inferred from the foregoing figure, or the last in Plate XXXII.
- 3. That the strength of a pillar, with moveable or rounded ends, is equal to that of a pillar of the same diameter, and double the length, the ends being flat or firmly fixed, may be inferred from the reasoning above; but it is *proved* experimentally from various results in the abstract p. 391 of the former research, or as below.
- 4. The strengths of pillars of the same diameter, but of different lengths, vary inversely as the 1.7th or 1.6th power of the length, or in this case as 2<sup>17</sup> or 2<sup>16</sup> nearly, according to my experiments. Hence the strength of a pillar one-half the length of another is 3.249 or 3.031 times greater than it; or in other words, the strength of a pillar with flat ends is somewhat more than three times as great as that of one of the same length and diameter with rounded ends.

 $_{3.077}$  \ Mean ratio 1:3.107.

Abstract from Tables VIII., IX., XII., XIII., &c. of the former Research on the results obtained from two hollow

		Pillars with rounded ends.					
	Length of pillar.	Diameter of section.	Number of experiments deduced from.	Breaking weight.	Length of pillar.		
	7 ft. $6\frac{3}{4}$ in., or $90\frac{3}{4}$ in.	External diameter 1.78 in. Internal diameter 1.21 in.	1	5585	7 ft. $4\frac{3}{4}$ in., or $88\frac{3}{4}$ in.		
Hollow columns of	90 <u>3</u> in.	External diameter 1.74 in. Internal diameter 1.187 in.	1	5711	88 <u>3</u> in.		
Low Moor iron, Y	90 <u>3</u> in.	External diameter 2.01 in. Internal diameter 1.415 in.	1	8357	88 <del>3</del> in.		
	90 <u>3</u> in.	External diameter 2·23 in. Internal diameter 1·54 in.	1	12389	88 <u>3</u> in.		
	10 ft. 2 in., or 122 in., in- cluding caps on the ends.	External diameter 3·555 in. Internal diameter 2·693 in.	1	42331 lbs., or 43537 lbs. for 10 ft. or 120 in.	120 in.		
Hollow columns of Low Moor iron, No. 2.	122 in., including caps on the ends.	External diameter 3·532 in. Internal diameter 2·659 in.	1	44683 lbs., or 45956 lbs. for 10 ft. or 120 in.	120 in.		
(Wrought \{	90 <u>3</u> in.	Diameter 1.017 in.	2	1808	90 <u>3</u> in.		
iron.	$60\frac{1}{2}$ in.	Diameter 1.015 in.	2	3938	$60\frac{1}{2}$ in.		
Solid pillars.  Timber.	60½ in.	Side of square 1•75 in.	2	3197 The second pillar was capped with iron at the ends, to prevent their being crushed.	60½ in.		

Strength of Pillars of Low Moor Iron, No. 3, and of other Materials, with the pillars in the present research.

 Diameter of section.	Number of experiments deduced from.	Breaking weight (B).	We	Weight which would crush the specimen (C).  Ratio of the strengths of pillars with both ends rounded and with both flat, the diameters being considered equal		h both and with iameters	$\frac{\mathbf{B}}{\mathbf{C}}$ .
External diameter			(	lbs.	-	)	
1.78 in. Internal diameter 1.21 in. Area of section 1.339 sq. in.	1	17840 lbs., or 17177 lbs. for length $90\frac{3}{4}$ in.	section	146975	1:3.075		·1237
External diameter 1.74 in. Internal diameter 1.187 in. Area of section 1.2712 sq. in.	1	16705 lbs., or 16084 lbs. for length 903/4 in.		139584	1:2.816	1:3.077	·115
External diameter 2.01 in. Internal diameter 1.415 in. Area of section 1.6005 sq. in.	1	28353 lbs., or 27299 lbs. for length 903/4 in.	Crushing weight per square inch of 109801 lbs. = 49.018 tons.	175742	1:3.266	Mean ratio 1:3.077	·155
External diameter 2.23 in. Internal diameter 1.54 in. Area of section 2.0431 sq. in.	1	40569 lbs., or 39061 lbs. for length 903/4 in.	Crush	224330	1:3.153		·174
External diameter 3.555 in. Internal diameter 2.693 in. Area of section 4.230 sq. in.	1	114479 lbs. =51·11 tons.	weight per square inch 8 lbs.=42.825 tons.	405774	1:2.629		•282
External diameter 3·532 in. Internal diameter 2·659 in. Area of section 4·245 sq. in. Weight of pillar 130 lbs.	1	110649 lbs. =49·39 tons.	Crushing weight p	407203	1:2.408		•272 This pillar had been split at one end in the first experiment, and was hooped.
Diameter 1.02	1	5280			1:2.920	၂ ဖွ	·
Diameter 1·02	1	12990			1:3.298	1:3.07	
Side of square ' 1•75 in.	4	9625	••	•••••	1:3.011	Mean ratio 1:3.076	

- 6. The great difference between the strength of a pillar with both ends rounded and both ends flat, is rendered obvious by their different modes of fracture, the former breaking in one place only, the middle; and the latter in three, at the middle and at each end.
- 7. In pillars with one end rounded and one end flat, fracture takes place at about one-third of the distance from the rounded end; and if the base of the pillar be enlarged with a disc, the distance of the place of fracture from the top of a pillar  $30\frac{1}{4}$  inches long is but ·28 inch from one-third of the whole length from the top, deduced from a mean of two experiments, in which one pillar broke at  $10\cdot42$  inches, and the other at  $10\cdot30$  inches (mean,  $10\cdot36$  inches) from the rounded end (see p. 439 of former research).
- 8. If the pillar had had both ends rounded, then the centre of fracture would have been at the middle, at b (first pillar, Plate XXXII.), or at half the height: in the cases before us, fracture took place at two-thirds of the height nearly, or at b (second pillar). Hence, comparing the distances Ab, Ab of the points of fracture from the rounded ends of two pillars of equal diameter and height, one with both ends rounded, and the other with one end rounded and one end flat, we have those distances as  $\frac{1}{2}$  to  $\frac{1}{3}$ , or as 3 to 2 nearly.
- 9. The relative strengths of the two pillars should therefore be inversely as the 1.7th or 1.6th powers of their lengths, Ab, Ab, or as 3.249:6.473, or 3.031:5.8, differing not widely from 1 to 2, as is shown by various experiments in Tables I., II., V., XII., XIII. of the former research.
- 10. Moreover, it will appear that in the curve indicating the form of flexure to the third pillar, Plate XXXII., the distance Abc will apply to the first pillar, Abcd to the second pillar, and bcdef to the third pillar; these distances being inversely as 2, 3, 4, the 1.7th powers of which are 3.249, 6.473, 10.556, and the 1.6th powers 3.031, 5.8, 9.19, or as 1, 2, 3, nearly.
- 11. Hence, of three pillars of the same length and diameter, one with both ends rounded, one with one end rounded and one end flat, and the other with both ends flat, the strengths are as 1, 2, 3.

The following facts, from the subsequent experiments, illustrate the last of the preceding figures. See also forms of fracture, Plate XXXIII.

In pillars of "Second London Mixture," 2 feet 6 inches long and  $1\frac{1}{2}$  inch diameter, fracture took place near to the middle and by breaking across near each end.

A pillar of Old Hill iron, 5 feet long and  $2\frac{1}{2}$  inches diameter, broke in the middle, and broke across near each end.

In pillars 6 feet 3 inches long, fracture took place in the middle, and each end became split. This was the case in all these pillars.

In pillars 7 feet 6 inches long and  $2\frac{1}{2}$  inches in diameter—that of Derwent iron broke in the middle and split off at each end; one of Old Park and one of London Mixture broke in three pieces, and split off at one end: there would appear to have been more points of contrary flexure in each of these two pillars than in the other cases.

#### General Remarks on the following Experiments.

The experiments forming the basis of this research are on the strength of cast-iron pillars in two series, the first being on Low Moor Iron, No. 2, each 10 feet long, varying in diameter from  $2\frac{1}{2}$  to 4 inches, solid and hollow; of these there are fourteen, including two on Blaenavon iron, No. 3; the second series are on solid pillars of cast iron, 10 feet long and  $2\frac{1}{2}$  inches diameter, from various parts of the kingdom, making twenty-four pillars, with two of a smaller diameter; other pillars cut out of the preceding ones, and made 7 feet 6 inches, 6 feet 3 inches, 5 feet, &c. long, amount with the former ones to seventy-two.

The experiments on these are distributed in Tables I. to VI. In two other Tables (VII. and VIII.) other experiments are given, to determine the transverse strengths of some of the irons, in order to compare these with their direct strength as pillars, besides numerous experiments to obtain the crushing strengths of the pillars, and the comparative hardness of the irons in different parts of their sections. The forms of the fractures of pillars in the first six Tables are given in Plate XXXIII.

It has been seen that the strength of long pillars of cast iron varies as a constant power n of the diameter nearly; the pillars being solid and uniform in texture, and nearly as the difference of the same powers of the external and internal diameters, when the pillars are hollow; or as  $d^n$  in solid pillars, and  $D^n - d^n$  in hollow ones, D and d being the external and internal diameters. This power n was found in my former researches\* to be 3.55, from the mean result of many experiments on the strength of pillars flat at the ends. In the following experiments, which are on pillars of a larger kind than before, and the iron therefore somewhat softer than iron cast in smaller masses, the average value of n is much more near to 3.50 than to 3.55, the mean from nineteen comparisons being 3.513 (page 865). The strength of pillars, whose diameter is the same, varied inversely as the 1.7th power of the length nearly, according to the average result from pillars with rounded ends and with flat ends, in my former experiments. But in the present ones, on flat-ended pillars only, the power of the length from the results of twenty-five comparisons (page 866) is 1.63 nearly, instead of 1.7, as before found.

The preceding results apply only to pillars whose length is thirty times the diameter or upwards; for pillars shorter than this, the formula for their strength would need correcting by the introduction of the crushing weight of the pillar.

Tables of the powers above for pillars likely to occur in practice are as below.

<sup>\*</sup> Philosophical Transactions, 1840.

Table I.—Powers of Diameters, or  $d^{3.5}$ .

$1.0^{3.5} = 1.0000$	$3.7^{3.5} = 97.433$	$5.7^{3.5} = 442.14$	$7.7^{3.5} = 1266.83$
1·25 <sup>3·5</sup> = 2·1837	$3.75^{3.5} = 102.12$	$5.75^{3.5} = 455.87$	$7.75^{3.5} = 1295.85$
$1.5^{3.5} = 4.1335$	3.83.5 = 106.965	5.83.5 = 469.89	$7.8^{3.5} = 1325.35$
$1.75^{3.5} = 7.0898$	$3.9^{3.5} = 117.15$	$5.9^{3.5} = 498.86$	$7.9^{3.5} = 1385.78$
$2 \cdot 0^{3 \cdot 5} = 11 \cdot 314$	$4 \cdot 0^{3 \cdot 5} = 128 \cdot 00$	$6 \cdot 0^{3 \cdot 5} = 529 \cdot 09$	$8.0^{3.5} = 1448.15$
$2 \cdot 1^{3 \cdot 5} = 13 \cdot 4205$	$4 \cdot 1^{3 \cdot 5} = 139 \cdot 55$	$6 \cdot 1^{3 \cdot 5} = 560 \cdot 60$	$8.25^{3.5} = 1612.83$
$2 \cdot 2^{3 \cdot 5} = 15 \cdot 7935$	$4\cdot2^{3\cdot5} = 151\cdot835$	$6 \cdot 2^{3 \cdot 5} = 593 \cdot 43$	$8.5^{3.5} = 1790.47$
2•25 <sup>3·5</sup> =17·086	$4.25^{3.5} = 158.26$	$6.25^{3.5} = 610.35$	8.753.5=1981.66
$2\cdot3^{3\cdot5}=18\cdot452$	$4.3^{3.5} = 164.87$	$6\cdot3^{3\cdot5} = 627\cdot61$	$9 \cdot 0^{3 \cdot 5} = 2187 \cdot 00$
$2 \cdot 4^{3 \cdot 5} = 21 \cdot 416$	$4 \cdot 4^{3 \cdot 5} = 178 \cdot 68$	$6 \cdot 4^{3 \cdot 5} = 663 \cdot 18$	9.253.5=2407.11
$2 \cdot 5^{3 \cdot 5} = 24 \cdot 705$	$4.5^{3.5} = 193.305$	$6.5^{3.5} = 700.16$	$9.5^{3.5} = 2642.61$
$2 \cdot 6^{3 \cdot 5} = 28 \cdot 340$	$4.6^{3.5} = 208.76$	$6 \cdot 6^{3 \cdot 5} = 738 \cdot 59$	9.753.5=2894.12
$2 \cdot 7^{3 \cdot 5} = 32 \cdot 3425$	$4.7^{3.5} = 225.08$	$6.7^{3.5} = 778.51$	$10 \cdot 0^{3 \cdot 5} = 3162 \cdot 28$
2.75 <sup>3.5</sup> =34.488	4.753.5=233.58	$6.75^{3.5} = 799.03$	$10.25^{3.5} = 3447.73$
$2 \cdot 8^{3 \cdot 5} = 36 \cdot 733$	$4 \cdot 8^{3 \cdot 5} = 242 \cdot 295$	$6.8^{3.5} = 819.94$	$10.5^{3.5} = 3751.13$
$2.9^{3.5} = 41.533$	$4.9^{3.5} = 260.43$	$6 \cdot 9^{3 \cdot 5} = 862 \cdot 92$	10.753.5=4073.14
$3.0^{3.5} = 46.765$	$5 \cdot 0^{3 \cdot 5} = 279 \cdot 51$	$7 \cdot 0^{3 \cdot 5} = 907 \cdot 49$	$11.0^{3.5} = 4414.43$
$3\cdot1^{3\cdot5}=52\cdot4525$	$5 \cdot 1^{3 \cdot 5} = 299 \cdot 57$	$7 \cdot 1^{3 \cdot 5} = 953 \cdot 68$	$11.25^{3.5} = 4775.66$
$3\cdot2^{3\cdot5} = 58\cdot617$	$5 \cdot 2^{3 \cdot 5} = 320 \cdot 635$	$7 \cdot 2^{3 \cdot 5} = 1001 \cdot 53$	$11.5^{3.5} = 5157.54$
3·25 <sup>3·5</sup> =61·886	5·25 <sup>3·5</sup> =331·56	$7.25^{3.5} = 1026.08$	$11.75^{3.5} = 5560.74$
$3.3^{3.5} = 65.283$	$5 \cdot 3^{3 \cdot 5} = 342 \cdot 74$	$7.3^{3.5} = 1051.07$	$12.0^{3.5} = 5985.96$
$3\cdot4^{3\cdot5} = 72\cdot473$	$5 \cdot 4^{3 \cdot 5} = 365 \cdot 91$	$7 \cdot 4^{3 \cdot 5} = 1102 \cdot 33$	
$3.5^{3.5} = 80.212$	$5.5^{3.5} = 390.18$	$7.5^{3.5} = 1155.35$	
$3\cdot6^{3\cdot5} = 88\cdot5235$	$5\cdot6^{3\cdot5} = 415\cdot58$	$7.6^{3.5} = 1210.17$	
	1		1

Table II.—Powers of Lengths, or  $l^{1.63}$ .

$1^{1\cdot 63} = 1\cdot$	$6\frac{1}{4}^{1\cdot63} = 19\cdot8282$	$12^{1\cdot63} = 57\cdot4203$	$19^{1.63} = 121.442$
$2^{1.63} = 3.0951$	$7^{1.63} = 23.8512$	$13^{1.63} = 65.4226$	$20^{1\cdot63} = 132\cdot032$
$2\frac{1}{2}^{1.63} = 4.4529$	$7\frac{1}{2}^{1\cdot63} = 26\cdot6901$	$14^{1\cdot 63} = 73\cdot 8225$	$21^{1\cdot63} = 142\cdot961$
$3^{1.63} = 5.9939$	$8^{1.63} = 29.6508$	$15^{1\cdot 63} = 82\cdot 6093$	$22^{1\cdot63} = 154\cdot223$
$4^{1.63} = 9.5798$	$9^{1.63} = 35.9265$	$16^{1\cdot 63} = 91\cdot 7731$	$23^{1\cdot63} = 165\cdot812$
$5^{1.63} = 13.7823$	$10^{1.63} = 42.6580$	$17^{1\cdot 63} = 101\cdot 305$	$24^{1\cdot63} = 177\cdot723$
$6^{1.63} = 18.5518$	$11^{1\cdot 63} = 49\cdot 8276$	$18^{1\cdot63} = 111\cdot197$	-

Further Remarks on the Experiments on the Strength of Pillars (Tables I. and II.).

If the strength of a pillar be represented by  $W=m \times \frac{D^n-d^n}{l^p}$ , where D, d are the external and internal diameters in inches, l the length in feet, m, n, p constant quantities, the following experiments (1st Summary) will furnish us with the value of

 $\frac{W}{D^n-d^n} = \frac{m}{l^p}$ , of which expressions the first is represented by x, and is calculated on two suppositions for the value of n; the other depends on the value of p. From my former experiments, n was taken as 3.55, and p as 1.7; and from the experiments in the present research, we have n=3.5, and p=1.63, as mentioned before. To assist in computing the strength of pillars, according to this last supposition, the two Tables of powers of the diameters and lengths of pillars previously given (page 860) have been formed. The experiments give the mean value of x, or  $\frac{W}{D^n-d^n}$  as 2085.2 or 2223.7 lbs. for the strength of a pillar 10 feet long and 1 inch diameter; but for a pillar of 1 foot long and 1 inch diameter it would be  $x \times 10^p$ , where  $10^p = 50.119$ , or 42.658.

#### First Summary of Results.

Results from the Experiments in Tables I. and II. upon pillars of Low Moor Iron, No. 2, and Blaenavon Iron, No. 3, the length of each being 10 feet, and the ends turned flat and perpendicular to the axis.

No. of experiment.		Value of $x$ , or the breaking weight of a solid pillar 10 feet long and 1 inch diameter, flat at the ends, from the formula $ \frac{W}{D^{3\cdot55}-d^{3\cdot55}} $		Value of $x$ from the formula $x = \frac{W}{D^{3.5} - d^{3.5}}.$		Error from using the mean from power 3.55, in terms of the value of x.	using the mean from power 3.5,
	- 1.	lbs. 2201·1		lbs. 2324•9		- <del>1</del> 19·3	-123
	2.	2332•9		2463.7		$-\frac{1}{9.4}$	1 10.3
	3.	2226•7		2374•2		- 1 15·7	<del>-1</del> 15.8
Hollow pillars, from $2\frac{1}{2}$ to 4 inches ex-	4.	2285•8		2471·3		$-\frac{1}{11\cdot 4}$	$-\frac{1}{10}$
ternal diameter	5.	1975.6	Low Moor, No. 2.	2121.7	Mean 2223•7	$+\frac{1}{18\cdot02}$	$+\frac{1}{20.8}$
	6.	2023-3	Mean 2085.2	2173.9	1/10uii WWW 1	$+\frac{1}{32.7}$	$+\frac{1}{43.7}$
	7.	1963•8		2127.6		$+\frac{1}{16\cdot 2}$	$+\frac{1}{22\cdot 1}$
l	8.	1785.3		1928.8		$+\frac{1}{5.95}$	$+\frac{1}{6.54}$
	11.	2008.9	`	2105.2		$+\frac{1}{26\cdot 3}$	$+\frac{1}{17.8}$
Solid pillars, $2\frac{1}{2}$ inches	12.	2048.5		2146•1 ∫		$+\frac{1}{55.8}$	$+\frac{1}{27.7}$
in diameter	13.	2034·1	Blaenavon, No.3.	2131.8	Mean 2164·4	$+\frac{1}{65}$	$+\frac{1}{65\cdot 4}$ .
	14.	2096.7	Mean 2065·4	2197.0		$-\frac{1}{67}$	$-\frac{1}{67\cdot 4}$

In the first two experiments of the preceding summary, the results are perhaps somewhat too high, though great care was taken to avoid error. In the columns (Experiments 7 and 8) they are too low. These had been weakened by being turned on the outside to reduce their strength, as they had been cast stronger than it was supposed the MDCCCLVII.

machinery would break without injury to it. The great reduction in strength produced by taking away the external crust, shows that to ornament a pillar it would not be prudent to plane it. It may, however, be mentioned that the eighth pillar had many flaws in it.

In experiments upon hollow pillars, it is frequently found that the metal on one side is much thinner than that on the other, but this does not produce so great a diminution in the strength as might be expected, for the thinner part of a casting is much harder than the thicker (Table VIII.), and this usually becomes the compressed side.

The strength of a solid pillar, 10 feet long and 1 inch diameter, being, from the first mean above,  $2085 \cdot 2$  lbs., that of a solid pillar, 1 foot long and 1 inch diameter, will, since  $10^{17} = 50 \cdot 119$ , be  $2085 \cdot 2 \times 50 \cdot 119 = 104508$  lbs., or  $46 \cdot 65$  tons; the strength being supposed to vary directly as the  $3 \cdot 55$  power of the diameter, and inversely as the  $1 \cdot 7$  power of the length, as obtained from the experiments in my former paper on this subject\*.

The larger and more varied pillars in the present research, have, by experiments deduced from most of the irons in the kingdom, given more precise results for the constants on which the strength depends. They show that it varies directly as the 3.5 power of the diameter, much more nearly than as the 3.55 power, and inversely as the 1.63 power of the length, from a mean of many experiments in each case (see pages 865 and 866).

From the second mean value of x in the preceding summary, that depending on the 3.5 power of the diameter, the strength of a pillar 10 feet long and 1 inch diameter is 2223.7 lbs., and this multiplied by  $10^{1.63}$  or 42.658 (see Table II. page 860), gives 94858.59 lbs.=42.347 tons, for the strength of a pillar 1 foot long and 1 inch diameter.

Whence w being the breaking weight in tons of a pillar whose length is l in feet, and external and internal diameters D and d in inches, the ends being flat and well-bedded, we have, in Low Moor iron, No. 2—

$$w=46.65 \times \frac{D^{3.55}-d^{3.55}}{l^{1.7}}$$
, from formulæ in Philosophical Transactions, 1840;  $w=42.347 \times \frac{D^{3.5}-d^{3.5}}{l^{1.63}}$ , from formulæ in the present paper.

The preceding chapter contains the results of experiments on the strength of pillars 10 feet long, solid as well as hollow, the hollow ones varying from  $2\frac{1}{2}$  to 4 inches external diameter, and the solid ones cast from models  $2\frac{1}{2}$  inches diameter. They are all of Low Moor iron, No. 2, except two, Nos. 13 and 14, of Blaenavon iron. In the previous research the iron used was that of Low Moor, No. 3, and there appears to be very little difference in their strengths, though the No. 2 seems to be rather the stronger. The experiments in Table IX. of the previous paper are on hollow pillars 1.75 inch external diameter and upwards; and on solid pillars of the same diameter nearly, the

length being 7 feet  $6\frac{3}{4}$  inches. To compare the results of this latter series of pillars with those of a larger kind in the present research, we will select three hollow pillars, among the least objectionable in the series, and two solid ones, and reduce their results to what they would have been if the pillars had been 1 inch diameter and 10 feet long.

Length.	Diameter.	Breaking weight.	Value of $x$ , or strength of a pillar 1 inch diameter and 10 feet long, from formula $x = \frac{W}{D^{3.55} - d^{3.55}} \times \frac{l^{1.7}}{10^{1.7}}.$	Value of $x$ , or breaking weight of a pillar 1 inch diameter and 10 feet long, from formula $x = \frac{W}{D^{3\cdot 5} - d^{3\cdot 5}} \times \frac{l^{1\cdot 63}}{10^{1\cdot 63}}.$
in. 9034	External diameter 1.75 in. Internal diameter 1.11 in.	lbs. 20957	lbs. 2230•7	lbs. 2352·8
Hollow pillars $\stackrel{\checkmark}{\downarrow}$ 88 $\frac{3}{4}$	External diameter 2.04 in. Internal diameter 1.46 in.	32413	2222.6 Mean 2166.5	2369·7 > Mean 2338·2
903	External diameter 2.01 in. Internal diameter 1.368 in.	30789	2046·3	2292-2
Solid pillars $\dots \begin{cases} 90\frac{3}{4} \\ 90\frac{3}{4} \end{cases}$	Diameter 1.76 in. Diameter 1.72 in.	23179 21995	1937.6 1994.9 Mean 1966.2	2032·5 2090·3 Mean 2061·4
	The length l is	aken in fe	eet =7.5625 feet.	

Pillars of Low Moor Iron, No. 3.

Comparing the results of the pillars in the abstract above, which are on Low Moor iron, No. 3, with those in the first Summary, which are on the same iron, No. 2, and taking the value of x as a mean from the first six hollow pillars, we have as below:—

On the first supposition, that is, using the power 3.55 of the diameter, and 1.7 of the length,—

From the first summary, 2028·7 lbs. for solid, and 2174·2 for hollow pillars. From the above abstract, 1966·2 lbs. for solid, and 2166·5 for hollow pillars.

On the second supposition, that is, using the power 3.5 of the diameter, and 1.63 of the length—

From the first summary, 2125.6 lbs. for solid, and 2321.6 for hollow pillars. From the above abstract, 2061.4 lbs. for solid, and 2338.2 for hollow pillars.

These results are nearly in accordance with one another, and they show in both series of experiments, and according to both suppositions, that the hollow cylinders are somewhat stronger than the solid ones; which is attributable, no doubt, to the superior hard-

ness of the metal in the outer ring of the hollow castings: on this subject we shall see more further on (page 867).

According to the first supposition, the values of x for a hollow pillar and for a solid pillar are nearly as 22 to 20, and on the second supposition, nearly as 23 to 21. Of the two irons, the No. 2 seems to be somewhat the stronger, as appears also from the next article.

Relative Strengths of solid Pillars of Low Moor Iron, No. 3, and those of No. 2 of the same Iron.

From the experimental researches\* we have for solid cylindrical pillars of Low Moor iron, No. 3, as a mean from ten experiments on small pillars,  $w=98922 \times \frac{d^{3\cdot55}}{l^{1\cdot7}}$ , where w is the breaking weight in lbs., d the diameter in inches, l the length in feet, 3·55 the power of the diameter, and 1·7 the inverse power of the length on which the strength depends, nearly, and 98922 lbs. the strength of a pillar 1 foot long and 1 inch diameter. To make the comparison above, we shall adduce the results of the pillars in Table II. Exp. 11, 12, and Table IV. Exp. 11 of this research, from which we obtain, by the same method of computation, for the strength of a pillar 1 foot long and 1 inch diameter of Low Moor iron, No. 2,—

			ibs.
From Table II. Exp. 11			100684
From Table II. Exp. 12		•	102668
From Table IV. Exp. 11			100518
$\mathbf{Mean}$			$\overline{101290}$

11

Whence the strengths of Low Moor iron, No. 3 and No. 2, are as 98922 to 101290, or as 1 to 1.02394, by which the strengths of the pillars of Low Moor iron, No. 3, must be multiplied to obtain those of No. 2.

Two other small solid pillars in the preceding abstract, compared with the three in the present, give 98544 to 101290, or 1 to  $1\cdot02786$ .

### Strength of Pillars dependent on their Lateral Dimensions.

In long pillars with their ends flat we have  $\dagger$ , in cylinders  $60\frac{1}{2}$  inches long, comparing the strength of that of

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·77 inch diameter with that of ·997 inch diameter, n=3.606
·77 inch diameter with that of 1·29 inch diameter, n=3.641
·77 inch diameter with that of 1·56 inch diameter, n=3.495
·997 inch diameter with that of 1·29 inch diameter, n=3.670
·997 inch diameter with that of 1·56 inch diameter, n=3.428
Mean . . n=3.568
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<sup>\*</sup> Philosophical Transactions, 1840, Part II. p. 402.

In pillars of the same length with discs at the ends, comparing

- ·51 inch diameter with 1·00 inch diameter, n=3.922
- ·51 inch diameter with 1·28 inch diameter,  $n=3\cdot820$
- ·51 inch diameter with 1·53 inch diameter, n=3.775
- ·775 inch diameter with 1·53 inch diameter, n=3.568
- ·775 inch diameter with 1·28 inch diameter, n=3.578
- 1.00 inch diameter with 1.53 inch diameter, n=3.412

Mean . . 
$$n=3.679$$

In the larger pillars of the present research, each 10 feet long, and cast from models varying from  $1\frac{1}{2}$  to  $2\frac{1}{2}$  inches diameter, the iron being that termed "Second London Mixture" (see Table III. Exp. 21 to 24), comparing the strengths of pillars of the following diameters, with their ends turned flat, we have

1.530 inch diameter with 2.511 inches diameter, n=3.5016

1.541 inch diameter with 2.511 inches diameter, n=3.6154

1.530 inch diameter with 2.496 inches diameter, n=3.3708

1.541 inch diameter with 2.496 inches diameter, n=3.4840

Mean . . . 
$$n=3.4929$$

In the same pillars rendered quite straight, and cut to be 6 feet 3 inches long, the ends being turned flat (see Table V. Exp. 5 to 9), we have, by comparing, as before,

1.530 inch diameter with 2.511 inches diameter, n=3.3068

1.541 inch diameter with 2.511 inches diameter, n=3.2603

1.530 inch diameter with 2.496 inches diameter, n=3.1670

1.541 inch diameter with 2.496 inches diameter, n=3.1179

Mean . . 
$$n=3.2130$$

Mean from all the above comparisons, nineteen in number, =3.513.

### Strength of Pillars dependent on their Length.

To find the inverse power of the length on which the strength of solid pillars of cast iron, cast from the same model, depends, the pillars being of various lengths and having their ends turned flat and perpendicular to the axis. From my former experiments \* we have as below, x being the inverse power, and the lengths of the pillars varying from eighty to twenty times the diameter nearly.

Diameters considered equal.	Lengths of which the strengths are compared.	Value of x.	Mean value of $x$ .
in. in.  •77 and •77  •77 and •777  •77 and •775  •997 and 1•01  •997 and 1•022	ins. ins. 60.5 and 30.25 60.5 and 20.1666 60.5 and 15.125 60.5 and 30.25 60.5 and 20.1666	1.843 1.682 1.550 1.691 1.483	1.650

<sup>\*</sup> Philosophical Transactions, 1840, Part II. p. 399.

Diameters of pillars compared.	pil	ths of lars pared.	Values of x.	Mean values of x.	Remarks.
in. in. $2\frac{1}{2}$ and $2\frac{1}{2}$ $2\frac{1}{3}$ and $2\frac{1}{3}$	ft. 10 10	ft. $7\frac{1}{2}$ $6\frac{1}{4}$	1.6303a 1.8864) b	1.6303	<sup>a</sup> This result is a mean from ten comparison on different kinds of iron. Tables II., III., IV
$2\frac{1}{2}$ and $2\frac{1}{2}$ $2\frac{1}{2}$ and $2\frac{1}{2}$ $2\frac{1}{2}$ and $2\frac{1}{2}$	10 10 10	$\begin{array}{c} 6\frac{1}{4} \\ 6\frac{1}{4} \\ 6\frac{1}{4} \end{array}$	1.5737 1.5514 1.4137	1.6063	b From four kinds of iron, Level, London Mixture, Old Hill and Derwent. Tables III
$2\frac{1}{2}$ and $2\frac{1}{2}$ $2\frac{1}{2}$ and $2\frac{1}{2}$ $1\frac{1}{2}$ and $1\frac{1}{2}$ $1\frac{1}{2}$ and $1\frac{1}{2}$	10 10 10 10	$6\frac{1}{4}$ $6\frac{1}{4}$ $6\frac{1}{4}$ $6\frac{1}{4}$	$ \begin{vmatrix} 1.482 \\ 1.4752 \\ 1.8338 \\ 1.8986 \end{vmatrix}^{c} $	1.6724	and V.  c From the Second London Mixture. Tables III. and V.
$2\frac{1}{2}$ and $2\frac{1}{2}$ $2\frac{1}{2}$ and $2\frac{1}{2}$	10 10	5 5	$egin{array}{c} 1 \cdot 4523 \ 1 \cdot 5942 \ \end{array}$	1.5232	d From Old Hill Iron. Tables III. and V

From the present experiments on pillars whose models were  $2\frac{1}{2}$  inches diameter in most cases, Tables I. to VI.

In the former experimental paper, mentioned above, I assumed 1.7 as the inverse power which the strength of pillars of the same diameter and of different lengths followed nearly, this applying both to pillars with rounded ends and to those with flat ends; but it appears from above, that in flat-ended pillars, representing the great mass of those used in practice, the strength would be better represented by some number intermediate between 1.6 and 1.7: the results of the experiments here detailed give 1.63 as a mean nearly, and this number may be used for all pillars whose length is more than about thirty times the diameter. In pillars shorter than this the formula used for calculating their strength would require correcting, as previously mentioned in page 859.

# Strength of Pillars, the Iron in the section of which is not uniform.

The iron of all the pillars in this research is assumed to be uniform and of equal resistance throughout the surface of fracture, but this is perhaps seldom or never the case in cast iron, except possibly in very large castings. My experiments on the crushing of cast iron, made for the British Association, to determine the relative values of hot and cold blast iron\*, of which an abstract is given in my 'Experimental Researches on the Strength and other properties of Cast Iron, 1846†, 'showed that small cylinders of cast iron, No. 2, from the Carron Iron-works, turned to the exact dimensions from castings made but little larger than the size intended, required to crush them 130163 lbs. per square inch when they were  $\frac{3}{8}$  inch diameter, from means of several experiments. In cylinders and prisms cut out of larger masses, the crushing weight per square inch was reduced to 119730 lbs., 103012 lbs., 102884 lbs., 108315 lbs., 96764 lbs., &c. In the experiments made to determine the crushing strength of the iron in my former experiments on pillars‡, cylinders  $\cdot$ 52 inch diameter and nearly of the size of the original casting, required 115910 lbs.

<sup>\*</sup> Seventh Report, 1837.

<sup>†</sup> WEALE, London.

<sup>‡</sup> Philosophical Transactions, 1840, Part II. p. 419.

per square inch to crush them, and small rectangles cut out of masses 1 inch square required 103692 lbs. only.

In some experiments made by Captain (now Colonel) James, as a Member of the Royal Commission for inquiring into the application of Iron to Railway Structures\*, it was found that the central part of bars of iron planed was much weaker to bear a transverse strain than bars cast of the same size. He states that "it was found by planing out  $\frac{3}{4}$ -inch bars from the centre of 2-inch square and 3-inch square bars, that the central portion was little more than half the strength of that from an inch bar, the relation being as 7 to 12" (there may be some doubt whether the iron was sound at the centre). In page 111 of the same Report, I showed that rectangular bars of cast iron, cast 1, 2, and 3 inches square, laid upon supports  $4\frac{1}{2}$  feet, 9 feet, and  $13\frac{1}{2}$  feet asunder, were broken by weights of 447 lbs., 1394 lbs., and 3043 lbs. respectively. These weights, divided by the squares of the lengths, should give equal results; the quotients, however, were as 447, 349, and 338 respectively. I attributed this falling off and deviation from theory partly to the defect of elasticity, which I had always found in cast iron, but principally to the superior hardness of the smaller castings.

From the experiments on the resistance to a crushing force of the various kinds of iron used in this research, it was found that short cylinders,  $\frac{3}{4}$  inch in diameter and  $1\frac{1}{2}$  inch high, cut out of pieces cast  $2\frac{1}{2}$  inches diameter, resisted with more force when cut out of the part near to the surface than when cut out of that near to the centre. To try this, many small cylinders, from four or five kinds of iron, were cut from the centre, and from the part intermediate between the centre and the circumference (Table VIII.), and it was found that the cylinders from the latter part were always stronger than those from the centre; thus, in cylinders from pillars of Old Park iron, No. 1, cast as above mentioned  $2\frac{1}{2}$  inches diameter, the mean crushing force of the iron near to the circumference was 39.32 tons per square inch, and that from the centre 33.33 tons per square inch, the difference being 5.99 tons. Whence  $\frac{5.99}{39.32}$ =·1523, the ratio of the falling off of the resistance of the central part to the resistance of the external part.

Cylinders from other irons used gave the results below (see Table VIII.).

Low Moor Iron, No. 2.—Crushing force per square inch.

Centre, 29.65 tons. Intermediate part, 34.59 tons.

External ring of a hollow pillar, 4 inches diameter, of which the outer crust had been removed, 39.06 tons.

External ring of a hollow pillar, about  $3\frac{1}{2}$  inch. diameter, 51.78 tons. From another pillar, of about the same diameter, 49.20 tons; the iron of both of these being very thin.

Blaenavon Iron, No. 3.

Centre, 34.22 tons. Intermediate part, 37.28 tons.

\* See Report, 1849, p. 250.

Second London Mixture.

Centre, 35.02 tons. Intermediate part, 42.56 tons.

Second London Mixture.—Specimens cut from pillars  $1\frac{1}{2}$  inch diameter.

Centre, 46·46 tons. Intermediate part, 49·59 tons.

By comparing the results of the experiments on the resisting powers of the different irons in the kingdom to a crushing force, with those obtained from the direct strengths of pillars of the same irons, 10 feet long and  $2\frac{1}{2}$  inches diameter, it appears that the crushing force varies rather widely from being as the direct strength of pillars from the same model. Though the ratio assumed is merely an approximation, it will afford means of approaching to the falling off in the strength of pillars, when the resistance of the different parts to a crushing force is known.

Taking the results from the breaking weights of three of the strongest iron pillars, each 10 feet long and  $2\frac{1}{2}$  inches diameter, and the crushing weights of the irons from those pillars, we have from the strongest pillars—

 $\begin{array}{c} \text{Breaking weights of the pillars.} & \text{Crushing weights of the iron in their sections.} \\ 66086 \\ 62794 \\ 61147 \end{array} \\ \text{Mean } 63342 \text{ lbs.} & 485750 \\ 61147 \end{array} \\ \text{and from those of the weakest irons---} \\ \begin{array}{c} 52680 \\ 49387 \\ 44918 \end{array} \\ \text{Mean } 48995 \text{ lbs.} & 338450 \\ 346130 \\ 273455 \end{array} \\ \text{Mean } 319345 \text{ lbs.} \\ 273455 \end{array}$ 

Comparing the breaking weights of the pillars with the crushing weights of the irons, we have 48995: 63342:: 319345: 412857.

The last number, 412857, ought to have been 464070 if the breaking weights had been as the crushing weights. But the breaking weight of pillars depends on the tensile and crushing weights conjointly, though principally upon the latter; the assumption of that proportion must be taken as an approximation only, as previously mentioned.

Let r, r' be the resistances to crushing per square inch of the parts near to the circumference and to the centre respectively; D, d the external diameter, and that which includes the softer part of the metal near to the centre. Then r-r'= the falling off per square inch in the resistance of the softer part below that of the external part.

Whence  $\frac{r-r'}{r}$  = the defect of resistance of the central part, the resistance of the external part being represented by unity.

If then w be the breaking weight of a pillar; m, n constants, and as before assumed for solid pillars of uniform texture,

$$w=mD^n$$
, . . . . . . . . . . . . . . . . (A.)

the length being constant, then

$$w = m \left( D^n - \frac{r - r'}{r} d^n \right) \quad . \quad (B.)$$

for pillars of variable texture, as above. If the pillar be hollow r'=0, and

as before found.

The falling off in the strength of solid pillars from that of hollow ones of the same external diameter, and of the same kind of iron, may be seen very evidently from the results of the experiments in Tables I. and II., and made the subject of further investigation, if the principles assumed be admitted.

Example 1. If in a hollow pillar, n, in equation C, be 3.5, and d be taken of different values as below, we shall have  $w=m(D^{3\cdot5}-d^{3\cdot5})$ ; and

if 
$$d = \frac{D}{4}$$
,  $w = m(D^{3\cdot5} - \cdot 0078 D^{3\cdot5}) = m \times \cdot 9922 D^{3\cdot5}$ ;  
if  $d = \frac{D}{2}$ ,  $w = m(D^{3\cdot5} - \cdot 0884 D^{3\cdot5}) = m \times \cdot 9116 D^{3\cdot5}$ ;  
if  $d = \frac{2D}{3}$ ,  $w = m(D^{3\cdot5} - \cdot 242 D^{3\cdot5}) = m \times \cdot 758 D^{3\cdot5}$ ;  
if  $d = \frac{3}{4}D$ ,  $w = m(D^{3\cdot5} - \cdot 365 D^{3\cdot5}) = m \times \cdot 635 D^{3\cdot5}$ ;  
if  $d = \frac{4}{5}D$ ,  $w = m(D^{3\cdot5} - \cdot 458 D^{3\cdot5}) = m \times \cdot 542 D^{3\cdot5}$ ;  
if  $d = \frac{5}{6}D$ ,  $w = m(D^{3\cdot5} - \cdot 528 D^{3\cdot5}) = m \times \cdot 472 D^{3\cdot5}$ .

Example 2. In the formula (B.), for metals of variable texture,  $w=m\left(D^{n}-\frac{r-r'}{r}d^{n}\right)$ ; and supposing, for example, as in Low Moor iron,  $\frac{r-r'}{r} = \frac{39-29\frac{1}{2}}{39} = \frac{1}{4}$  nearly, or  $w = m \left( D^{3\cdot 5} - \frac{1}{4} d^{3\cdot 5} \right),$ 

if 
$$d = \frac{\mathbf{D}}{4}$$
,  $w = m \left( \mathbf{D}^{3\cdot5} - \frac{1}{4} \times \cdot 00781 \, \mathbf{D}^{3\cdot5} \right) = m \left( \mathbf{D}^{3\cdot5} - \cdot 0019 \, \mathbf{D}^{3\cdot5} \right) = m \times \cdot 998 \, \mathbf{D}^{3\cdot5} = 22 \cdot 954 \, \mathbf{D}^{3\cdot5};$  if  $d = \frac{\mathbf{D}}{2}$ ,  $w = m \left( \mathbf{D}^{3\cdot5} - \frac{1}{4} \times \cdot 0884 \, \mathbf{D}^{3\cdot5} \right) = m \left( \mathbf{D}^{3\cdot5} - \cdot 0221 \, \mathbf{D}^{3\cdot5} \right) = m \times \cdot 9779 \, \mathbf{D}^{3\cdot5} = 22 \cdot 492 \, \mathbf{D}^{3\cdot5};$  if  $d = \frac{2\mathbf{D}}{3}$ ,  $w = m \left( \mathbf{D}^{3\cdot5} - \frac{1}{4} \times \cdot 242 \, \mathbf{D}^{3\cdot5} \right) = m \left( \mathbf{D}^{3\cdot5} - \cdot 0605 \, \mathbf{D}^{3\cdot5} \right) = m \times \cdot 9395 \, \mathbf{D}^{3\cdot5} = 21 \cdot 608 \, \mathbf{D}^{3\cdot5};$  if  $d = \frac{3\mathbf{D}}{4}$ ,  $w = m \left( \mathbf{D}^{3\cdot5} - \frac{1}{4} \times \cdot 365 \, \mathbf{D}^{3\cdot5} \right) = m \left( \mathbf{D}^{3\cdot5} - \cdot 091 \, \mathbf{D}^{3\cdot5} \right) = m \times \cdot 909 \, \mathbf{D}^{3\cdot5} = 20 \cdot 907 \, \mathbf{D}^{3\cdot5};$  if  $d = \frac{4\mathbf{D}}{5}$ ,  $w = m \left( \mathbf{D}^{3\cdot5} - \frac{1}{4} \times \cdot 458 \, \mathbf{D}^{3\cdot5} \right) = m \left( \mathbf{D}^{3\cdot5} - \cdot 1145 \, \mathbf{D}^{3\cdot5} \right) = m \times \cdot 8855 \, \mathbf{D}^{3\cdot5} = 20 \cdot 366 \, \mathbf{D}^{3\cdot5};$ 

The mean value of x obtained from the first six pillars in the summary (page 861) is 2174 lbs., the power being 3.55; and for solid pillars it is 2029 lbs. (Exp. 11 and 12). These values for a power 3.5, as assumed in the problem above, are 2322 and 2126 lbs. 5 x MDCCCLVII.

respectively, or as 23 to 21 nearly; if then m be assumed as 23, the falling off for different values of d will be, as in the example above, becoming successively 22·954, 22·492, 21·608, 20·907, 20·366; and the value of d, which gives 21, is about  $\frac{3}{4}$ D.

Example 3. Again, suppose  $\frac{r-r'}{r} = \frac{1}{5}$ , and the rest as before,

if 
$$d = \frac{2}{3}D$$
,  $w = m\left(D^{3\cdot5} - \frac{1}{5} \times \cdot 242 D^{3\cdot5}\right) = m(D^{3\cdot5} - \cdot 048 D^{3\cdot5}) = m \times \cdot 952 D^{3\cdot5} = 21 \cdot 896 D^{3\cdot5};$  if  $d = \frac{3}{4}D$ ,  $w = m\left(D^{3\cdot5} - \frac{1}{5} \times \cdot 365 D^{3\cdot5}\right) = m(D^{3\cdot5} - \cdot 073 D^{3\cdot5}) = m \times \cdot 927 D^{3\cdot5} = 21 \cdot 321 D^{3\cdot5};$  if  $d = \frac{4}{5}D$ ,  $w = m\left(D^{3\cdot5} - \frac{1}{5} \times \cdot 458 D^{3\cdot5}\right) = m(D^{3\cdot5} - \cdot 0916 D^{3\cdot5}) = m \times \cdot 9084 D^{3\cdot5} = 20 \cdot 893 D^{3\cdot5};$  if  $d = \frac{4}{5}D$ ,  $d = m\left(D^{3\cdot5} - \frac{1}{5} \times \cdot 458 D^{3\cdot5}\right) = m(D^{3\cdot5} - \cdot 0916 D^{3\cdot5}) = m \times \cdot 9084 D^{3\cdot5} = 20 \cdot 893 D^{3\cdot5};$  and the value of  $d$ , which gives 21, is about  $\frac{4}{5}D$ .

The pillars in the following summary of results were all cast solid from models  $2\frac{1}{2}$  inches diameter, and made 10 feet long, with their ends flat. Those of iron, denominated No. 1, were chosen for comparison from some of the leading irons in the kingdom. They vary in strength from 29.50 tons to 20.05 tons, or nearly as 3 to 2. Formulæ for the strength of pillars of all the irons tried, thirteen in number, are given, both on the supposition used in the former research, and also on that arrived at in the present. The weights which would crush short pillars of the same diameter are likewise given.

#### Second Summary.

Results of all the Experiments in Tables II. and III. on Solid Pillars of Cast Iron, 10 feet long, cast from models  $2\frac{1}{2}$  inches diameter, and supplied from Low Moor, and from Messrs. Easton and Amos; with applications to the formulæ for their strength on different suppositions; and also the weights which would crush the pillare.

Description of Iron of which the pillar was formed.	Breaking weight of pillar.	Value of $x$ from formula $x = \frac{W}{D^{3.55}}$ , where $W$ is the breaking weight, $D$ the diameter, and $x$ the breaking weight of a pillar 1 inch diameter and 10 feet long.	Value of $x$ from formula $x = \frac{W}{D^{3\cdot 5}}$ , where $W$ is the breaking weight, $D$ the diameter, and $x$ the breaking weight of a pillar 1 inch diameter and 10 feet long.	Weight which would crush the pillar, calculated from Table VIII.
Old Park Iron, No. 1. Stourbridge. Cold blast.	$     \begin{array}{c}                                     $	$egin{array}{c}  ext{lbs.} \ 2571 \cdot 5 \ 2436 \cdot 8 \ \end{array}  ight\} 2504 \cdot 1$	$egin{array}{c}  ext{lbs.} \ 2692 \cdot 2 \ 2552 \cdot 3 \end{array} \Big\} \ 2622 \cdot 2$	$\begin{array}{c} {}^{\mathrm{lbs.}} \\ 433350 \\ 441350 \end{array} \right\}  437350$
Derwent Iron, No. 1. Durham. Hot blast.	$57855 \\ 67733 $ $62794$	$2147.2 \\ 2556.6$ 2351.9	$\left\{ egin{array}{c} 2249.2 \\ 2677.4 \\ \end{array}  ight\} 2463.3$	$\left\{ 488060 \atop 483440 \right\} 485750$
Portland Iron, No. 1. Tovine, Scotland. Hot blast.	$59736 \atop 62559$ $61147$	$\left\{ egin{array}{c} 2332.8 \ 2328.3 \end{array}  ight\}  2330.5$	2441·8 2438·8 } 2440·3	$\left. \begin{array}{c} 462760 \\ 475460 \end{array} \right\}  469110$
Calder Iron, No. 1. Lanarkshire. Hot blast.	$\left\{ \begin{array}{c} 60677 \\ 60677 \end{array} \right\} 60677$	2379·7 2283·8 } 2331·7	$\left\{ \begin{array}{c} 2490.7 \\ 2391.8 \end{array} \right\} 2441.2$	$\left\{ \begin{array}{c} 412200 \\ 421850 \end{array} \right\}  417025$
London Mixture. One-half old plate iron and one-half Calder Iron.	$54562 \\ 59266 $ $ 56914 $	$\begin{bmatrix} 2013.7 \\ 2141.9 \end{bmatrix} 2077.8$	$ \begin{vmatrix} 2109.4 \\ 2244.4 \end{vmatrix} 2176.9 $	$\left\{ egin{array}{c} 407790 \\ 412630 \end{array}  ight\} \left\{ 410210 \right]$
Level Iron, No. 1. Staffordshire. Hot blast.	$54091 \atop 56443$ $55267$	$2038.8 \atop 2179.2$ 2109.0	$2135 \cdot 2 \\ 2281 \cdot 5$ $2208 \cdot 3$	$\left\{ \begin{array}{c} 339700 \\ 335130 \end{array} \right\}$ $\left\{ \begin{array}{c} 337415 \end{array} \right]$
Coltness Iron, No. 1. Edinburgh. Hot blast.	$56443 \atop 48917$ $52680$	$2133.5 \\ 1902.1$ $2017.8$	$\left\{ \begin{array}{c} 2234.2 \\ 1991.1 \end{array} \right\} 2112.6$	$\left\{ \begin{array}{c} 313460 \\ 308505 \end{array} \right\}  310980$
Carron Iron, No. 1. County of Stirling. Hot blast.	$52210 \atop 53151$ $52680$	$2001.6 \\ 2026.1$ 2013.8	$2095.7 \\ 2121.6$ 2108.6	$\left\{ rac{337910}{338990}  ight\} 338450$
Blaenavon Iron, No. 1. South Wales. Cold blast.	$\left\{ egin{array}{c} 48917 \\ 49858 \end{array} \right\} 49387$	$1926.7 \\ 1930.5$ $1928.6$	$2016.5 \\ 2020.9$ $2018.7$	$\left\{ \begin{array}{c} 344460 \\ 347800 \end{array} \right\}  346130$
Old Hill Iron. No. 1. Staffordshire. Cold blast.	$\left\{ \begin{array}{c} 44683 \\ 45154 \end{array} \right\}  44918$	$1679.5 \atop 1690.0$ $1684.7$	$1759.0 \atop 1770.0$ $1764.5$	$273130 \ 273780$ $273455$
Second London Mixture. One-third No. 1 best Scotch pig iron, and two-thirds old metal.	$63499 \atop 58325$ $60912$	$\begin{bmatrix} 2417.2 \\ 2268.0 \end{bmatrix} 2342.6$	$ \begin{vmatrix} 2531 \cdot 1 \\ 2374 \cdot 1 \end{vmatrix} 2452 \cdot 6 $	$\left\{ \begin{array}{c} 472120 \\ 466490 \end{array} \right\}  469305$
Low Moor Iron, No. 2. Yorkshire. Cold blast.	$\begin{bmatrix} 55973 \\ 55973 \end{bmatrix} 55973$	$\begin{bmatrix} 2008.9 \\ 2048.5 \end{bmatrix} 2028.7$	$\begin{bmatrix} 2105.2 \\ 2146.1 \end{bmatrix} 2125.6$	$\begin{vmatrix} 397590 \\ 393240 \end{vmatrix}$ 395415
Blaenavon Iron, No. 3. South Wales. Cold blast.	$56914 \atop 57855$ 57384	$2034.1 \ 2096.7$ $2065.4$	$2131.8 \ 2197.0$ $2164.4$	$\left\{ \begin{array}{c} 428540 \\ 425190 \end{array} \right\}  426865$

Formulæ for computing the Breaking Weights of Solid Pillars of the Irons in the preceding summary; where W is the breaking weight in lbs. or in tons, D the diameter in inches, and l the length in feet.

Description of Iron in the Pillar.	Strength of a pillar from formula $W = m \cdot \frac{D^{3 \cdot 55}}{l^{1 \cdot 7}}$ , where $m$ is as below, according as the weight is in lbs or tons.	Strength of a pillar from formula $W=m.\frac{D^{3\cdot 5}}{l^{1\cdot 63}},$ where $m$ is as below, according as the weight is in lbs. or tons.
Old Park Iron, No. 1 Derwent Iron, No. 1 Portland Iron, No. 1 Calder Iron, No. 1 London Mixture Level Iron, No. 1 Coltness Iron, No. 1 Carron Iron, No. 1 Blaenavon Iron, No. 1 Old Hill Iron, No. 1 Second London Mixture Low Moor Iron, No. 2 Blaenavon Iron, No. 3	lbs. tons. $125502 = 56 \cdot 03$ $117872 = 52 \cdot 62$ $116802 = 52 \cdot 14$ $116862 = 52 \cdot 17$ $104137 = 46 \cdot 49$ $105700 = 47 \cdot 19$ $101130 = 45 \cdot 15$ $100929 = 45 \cdot 06$ $96659 = 43 \cdot 15$ $84435 = 37 \cdot 69$ $117408 = 52 \cdot 41$ $101676 = 45 \cdot 39$ $103515 = 46 \cdot 21$	$\begin{array}{c} \text{lbs.} & \text{tons.} \\ 111858 = 49 \cdot 94 \\ 105079 = 46 \cdot 91 \\ 104098 = 46 \cdot 47 \\ 104137 = 46 \cdot 49 \\ 92862 = 41 \cdot 46 \\ 94202 = 42 \cdot 05 \\ 90119 = 40 \cdot 23 \\ 89949 = 40 \cdot 16 \\ 86114 = 38 \cdot 44 \\ 75270 = 33 \cdot 60 \\ 104623 = 46 \cdot 21 \\ 90674 = 40 \cdot 48 \\ 92329 = 41 \cdot 22 \\ \end{array}$

The numbers above represent the strength of a pillar 1 foot long and 1 inch diameter.

To compare the direct strength of long pillars of cast iron with their transverse strength.—If the pillars be cylindrical, and have their ends turned flat and perpendicular to their axes, the ultimate strength of a solid pillar will be represented by  $w=m\frac{D^n}{l^p}$ , where D is the diameter, l the length, and m, n, p constants. My former experiments\* gave n=3.55 and p=1.7, but the present experiments on larger pillars give 3.5 and 1.63 for their values much more nearly.

The transverse strength of a cylinder, laid with its ends upon supports, and broken by a weight at its middle, is

$$w'=b.\frac{D^3}{I}$$

Whence the ratio of the direct strength of a pillar to its transverse strength is

$$\frac{w}{w'} = \frac{m}{b} \cdot \frac{D^{n-3}}{l^{p-1}}.$$

If n be taken as 3.55 and p as 1.7, we shall have

$$\frac{w}{w'} = \frac{m}{h} \cdot \frac{D^{55}}{77};$$

and if n be taken as 3.5 and p as 1.63,

$$\frac{w}{w'} = \frac{m}{b} \cdot \frac{\mathbf{D}^{5}}{l^{63}}.$$

\* Philosophical Transactions, 1840.

Tables III., IV. and VII. give both the direct and transverse strength of the pillars to which the preceding remarks are applicable.

The following summary contains the results of the experiments in Tables II. to VI. upon solid pillars varying in length from 10 feet to 7 feet 6 inches, 6 feet 3 inches, and 5 feet, or as 8, 6, 5, 4, all from models  $2\frac{1}{2}$  inches diameter; the results are from thirteen kinds of cast iron, as before, or from twenty-six pillars 10 feet long, eleven pillars 7 feet 6 inches long, six 6 feet 3 inches long, and three 5 feet long, forty-six pillars in all. This does not include pillars round at the ends, and others cast  $1\frac{1}{2}$  inch diameter.

The mean crushing strength of short pillars of the irons, cast  $2\frac{1}{2}$  inches diameter, is 175.7 tons. The other mean values are as below:—

Length.	Breaking weight.	Ultimate decrement of length.	Ultimate deflection.	Ratio of breaking weight to crushing weight.
10 ft. 7 ft. 6 in. 6 ft. 3 in. 5 ft.	tons. 23·27 40·57 51·77 57·69*	in. •176 •235 •248 •296*	in. •75 •66  •57*	·146 ·231 ·294 ·472*

whence it appears that the ultimate decrement of length is inversely as the length nearly. Another object of these experiments, and of others on pillars of smaller diameter in Tables III. and V., was to determine the powers of the diameter and length on which the strength of pillars depends (see pp. 865 and 866).

<sup>\*</sup> From two experiments only.

Third Summary.—Comparison of the Crushing Weights, Breaking Weights, ultimate diameter, the lengths being different, and the

	Weight which would crush a short pillar of	Pillars 10 fee	et long, fro	om Tables II. and III	
Description of Iron in the Pillar.	the iron used, and of the same diameter, 2½ inches (from Table VIII.). (C.)	Breaking weight. (B.)	<u>₽</u> .	Ultimate decrement of length.	Ultimate deflection.
Old Park Iron, No. 1. Cold blast	lbs. 437350	$\begin{array}{c} \text{lbs.} & \text{Mean.} \\ 65381 & 66086 \text{ lbs.} \\ 66792 & = 29.50 \text{ tons.} \end{array}$	·151	·181 ·171}·176	·31 ·45 } ·38
Derwent Iron, No. 1. Hot blast	485750	$57855$ $\left.\begin{array}{c} 62794 \text{ lbs.} \\ 67733 \end{array}\right\} = 28.03 \text{ tons.}$	•129	$\binom{164}{180}$ ·172	·58 ·69 } ·63
Portland Iron, No. 1. Hot blast	469110	59736 61147 lbs. $62559$ = $27.30$ tons.	·130	·175 ·179 } ·177	$\begin{bmatrix} .84 \\ .92 \end{bmatrix}$ .88
Calder Iron, No. 1. Hot blast	417025	$60677$ $\left. \begin{array}{c} 60677 \text{ lbs.} \\ 60677 \end{array} \right\} = 27 \cdot 09 \text{ tons.}$	·145	·177 ·192}·184	·45 1·13 } ·79
London Mixture. One-half old plate iron and one-half Calder Iron	410210	59266 $56914$ lbs. $54562$ $=25.41$ tons.	·139	·190 ·188}·189	1.06
Level Iron, No. 1. Hot blast	337415	$54091$ $\left.\begin{array}{c} 55267 \text{ lbs.} \\ 56443 \end{array}\right\} = 24.67 \text{ tons.}$	·164	·199 ·179}·189	1.14 .91 } 1.03
Coltness Iron, No. 1. Hot blast	310980	$56443$ $\left.\begin{array}{c} 52680 \text{ lbs.} \\ 48917 \end{array}\right\} = 23.52 \text{ tons.}$	·169	${}^{\cdot 163}_{\cdot 155}$ $\Big\} \cdot 159$	·57 ·44 } ·51
Carron Iron, No. 1. Hot blast	338450	$53151$ $\left.\begin{array}{c} 52680 \text{ lbs.} \\ 52210 \end{array}\right\} = 23.52 \text{ tons.}$	·156	·166 ·173}·169	$\begin{bmatrix} \cdot 38 \\ \cdot 45 \end{bmatrix}$ $\cdot 42$
Blaenavon Iron, No. 1. Cold blast	346130	$48917$ $\left.\begin{array}{c} 49387 \text{ lbs.} \\ 49858 \end{array}\right\} = 22.05 \text{ tons.}$	·143	·169 ·159}·164	·83 ·65 } ·74
Old Hill Iron, No. 1. Cold blast	273455	$44683$ \ $44918$ lbs. $45154$ \} = $20.05$ tons.	·164	·191 ·181 } ·186	·66 .61 ·64 .
Second London Mixture. One-third No. 1 best Scotch pig-iron, and two-thirds of old metal	469305	63499 $60912$ lbs. $58325$ $=27.19$ tons.	•130	·189 ·159}·174	·66 ·70} ·68
Low Moor Iron, No. 2. Cold blast	395415	$55973$ $\left.\begin{array}{c} 55973 \text{ lbs.} \\ 55973 \end{array}\right\} = 24.99 \text{ tons.}$	•141		1·23 ·79} 1·01
Blaenavon Iron, No. 3. Cold blast	426865	56914 $57384$ lbs. $57855$ $= 25.62$ tons.	·134		1.00
Means from all the results in each vertical column $\ldots$	393650 =175.74 tons.	52832 = 23.27  tons.	·146	·176	·75

Deflections, and ultimate Decrements of Length of the Cast Iron Pillars,  $2\frac{1}{2}$  inches ends turned flat and perpendicular to the axis.

Pillars 7 ft. 6 ir	ı. long,	from Ta	ble IV.	Pillars 6 feet	3 inches long,	from Table V.		Pillars 5	feet long, from	n Table VI.	
Breaking weight. (B.)	$\frac{\mathbf{B}}{\mathbf{C}}$ .	Ulti- mate decre- ment of length.	Ulti- mate deflec- tion.	Breaking weight. (B.)	• <u>B</u> .	Ultimate decrement of length.	Ulti- mate deflec- tion.	Breaking weight. (B.)	$\frac{\mathbf{B}}{\mathbf{C}}$ .	Ultimate decrement of length.	Ultimate deflection.
100191 lbs. =44.73 tons.	·229	in. •228	in. •56			in.		·			
105365 lbs. =47.04 tons.	·217	·225	·51	112438 lbs.* = 50·20 tons.	·231	•221					
102543 lbs. = 45.78 tons.	· <b>21</b> 9	·245	· <b>6</b> 9								
93135 lbs. =41.58 tons.	·223	·231	· <b>6</b> 8								
93605 lbs. =41.79 tons.	·228	·234	·67	114320  lbs. = $51.04 \text{ tons.}$	·279	· <b>277</b>					
98309 lbs. = 43.89 tons.	-291	•244	·65	131275 lbs. = 58.60 tons.	•389	·272					
87019 lbs. = 38.85 tons.	·280	·254	.92								
72437  lbs. = $32.34 \text{ tons.}$	.214	·235	·76								
71967 lbs. =32·13 tons.	·208	.222	∙59	••••		•••		117612 lbs.† =52·51 tons.	·339	·2 <b>5</b> 3	
			•••••	93622 lbs. = 41.79 tons.	·342	•264		$ \begin{array}{c c} 135592 \\ 122863 \\ \hline \text{Mean } 129227 \text{ lbs.} \\ = 57.69 \text{ tons.} \end{array} $	·496 ·449 } ·472	·285 ·308 } ·296	$\begin{bmatrix} .50 \\ .64 \end{bmatrix} .57$
				$ \begin{array}{c} 127432 \\ 116672 \\ \end{array} $ Mean 122052 lbs. $= 54 \cdot 49 \text{ tons.}$	$\begin{vmatrix} \cdot 272 \\ \cdot 249 \end{vmatrix} \cdot 260$	$\begin{bmatrix} \cdot 244 \\ \cdot 213 \end{bmatrix} \cdot 228$					
89371 lbs. =39.9 tons.	-226		-63								
85608 lbs. = 38.22 tons.	•201		-59								
90868 lbs. =40.57 tons.	·231	.235	•66	115960 lbs. =51.77 tons.	·294	-248					

<sup>\*</sup> This bar had probably been reduced in strength by straightening after a former experiment upon it.

The results hitherto given are principally deduced from the experiments in the eight following Tables.

<sup>†</sup> This result is probably less than it ought to be, as too large a weight was laid on at once.

Table I.—Experiments to obtain the Strength of Hollow Uniform Pillars of Low Moor Iron, No. 2, each pillar being 10 feet long, and having its ends turned flat and perpendicular to the axis.

			1			1	
No. of Experiment.	Description of Pillar. 10 feet long, and as above.	Deflection.	Weight producing the de- flection.	Breaking weight, or that with which the pillar sank.	Value of $x$ from formula $x = \frac{W}{D^{3\cdot55} - d^{3\cdot55}},$ where $W$ is the breaking weight, $D$ and $d$ the external and internal diameters, the length of the pillar being 10 feet.	Value of $x$ from formula $x = \frac{W}{D^{3\cdot 5} - d^{3\cdot 5}}$	Remarks.
1.	Hollow uniform cylinder.  External diameter by various measurements }  Internal diameter by various measurements }  Yeight of pillar 603 lbs.	·35 ·40	lbs. 20339 22635 24931 27227 29523 31819 34115	36411†, or 33197 Mean 34804 = 15·54 tons.	1bs. 2201·1	lbs. 2324·9	†It bore this nearly, but broke with 33197 lbs. laid on again.  For the form of fracture of this and the next pillar see Plate XXXIII.
2.	$ \begin{array}{ll} \mbox{Hollow uniform cylinder.} \\ \mbox{External diameter by six} \\ \mbox{measurements} \\ \mbox{Internal diameter by eight} \\ \mbox{measurements} \\ \mbox{Weight of pillar } 64\frac{1}{2} \mbox{ lbs.} \end{array} $	18 25 34 39 48 61 75 109 1.16*	20339 24931 27227 29523 31819 34115 36411 38707 39166	39166 = 17·48 tons.	2332·9	2463.7	*The deflections marked thus in this and the following experiments were computed from the two preceding results.
3.	Hollow uniform cylinder. External diameter from six measurements $3.021$ Internal diameter from ten measurements $2.354$ Weight of pillar $89\frac{1}{2}$ lbs.	·04 ·06 ·08 ·13 ·16 ·22 ·30 ·38 ·55 ·93	20339 24931 29523 38707 43299 47891 52483 57075 61667 66259	66259† =29·58 tons.	2226·7	2374·2	†With this weight it broke, after bearing it several minutes. The thickness of the ring of metal on the convex side was '55 and on the concave '17. The fracture took place nearly in the middle.
4.	Hollow uniform cylinder.  External diameter from nine measurements } 3.035  Internal diameter from ten measurements } 2.354  Weight of pillar 88 lbs.	·06 ·10 ·15 ·17 ·23 ·31 ·55 ·91	20339 24931 38707 43299 47891 61667 66259 68555	72228†, or 67636 Mean 69932 =31.22 tons.	2285·8	2471:3	rIt bore this nearly, but broke with 67636 lbs. laid on again.  The pillar broke nearly in the middle, and split at both ends; the metal differed in thickness on the opposite sides as 2 to 1 nearly, the thicker part being on the concave side and therefore compressed. See Plate XXXIII.
5.	Hollow uniform cylinders.  External diameter from six measurements	·32 ·42 ·55 ·84 1·00*	48917 58325 77141 86549 95957 105365 110649	110649 =49·39 tons.	1975-6	2121.7	With the weight 110649 this pillar sank down, but was not allowed to bend so as to break. It was preserved in order that an experiment might be made upon it in another way. When unloaded it became nearly straight, and was easily rendered so.
6.	Hollow uniform cylinder.  External diameter from eight measurements } 3.555  Internal diameter from eight measurements } 2.664  By computation	·19 ·23 ·31 ·45 ·63	58325 67733 77141 86549 95957 105365 110069 114479	114479 =51·11 tons.	2023-3	2173-9	The remark in Experiment 5 applies also to the weight 114479 with which this pillar sank.

Table I. (continued).

No. of Experiment.		Description of Pillar, feet long, and as above.	Deflection.	Weight producing the de- flection.	Breaking weight, or that with which the pillar sank.	Value of $x$ from formula $\frac{w}{x=\overline{D^{3\cdot55}-d^{3\cdot55}}}$ where W is the breaking weight, D and $a$ the external and internal diameters, the length of the pillar being 10 feet.	Value of $x$ from formula $x = \frac{W}{D^{3\cdot 5} - d^{3\cdot 5}}$	Remarks.
·	long. Externa twelve Internal eight	uniform cylinder 9 ft. 9 in. inches. diameter from measurements diameter from measurements 3.804 3.17 of pillar 106 lbs. 10 oz.	inch03 -04 -07 -10 -29 -48*	1bs. 20693 46271 74495 93311 102719 112127	112127, or 107403 lbs. =47.95 tons, if the pillar had been 10 feet long.	lbs. 1963·8	2127·6	This and the following pillar were cast to be 4 inches external diameter, and were found so strong as to require weights which would injure the machinery. They were therefore turned outside to reduce their strength, and this, by taking away the hard external surface of the metal, seems to have produced that effect in a very great degree. They were cast 9 feet 9 inches long instead of 10 feet, so that they might be broken with rounded caps on the ends, which would only have required one-third of the weight required by pillars with flat ends.  Ratio of thicknesses of metal on opposite sides in middle '37 to '25.
8.	long. Externa twelv Interna dle l ment	uniform cylinder 9 ft. 9 in  l diameter from e measurements  diameter at mid- y eight measure- s	·05 ·09 ·18 ·24	20693 39509 58325 77141 86549 99720	99720, or 95519 lbs. =42.64 tons if the pillar had been 10 feet long		1928-8	Ratio of thicknesses of metal on opposite sides in middle 21 to 39, the thin side being the concave one.  This pillar had many flaws in it.
9.	ed caps on the ends.	ollow pillar, the same as in Experiment 5. Ingth, including caps on the ends, 10 ft. 2 in.	·18	20693 25397 30101 32453 34805 37157 39509 41861 44213	44683, or 45956 lbs. = 20-51 tons the breaking weight of th pillar if 10 feet long	e		
10.	II Le	ollow pillar, the same as i Experiment 6. ength, including caps o the ends, 10 ft. 2 in.	.21	20693 25397 30101 34805 37157 39509 41861		s. r		

The hollow pillars in Experiments 5 and 6 had been prevented from being broken by a prop, which was made to act on their middle immediately after they had been bent beyond their utmost power of resisting. They were then unloaded and reserved for another experiment, as below; after being rendered straight and turned afresh at the ends, rounded caps were affixed to the ends, to enable the straining force to be applied 5 Y

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in the direction of the axis, as in Experiments 9 and 10. The strength in these cases should have been only one-third of what it was in the former ones, according to the results in pages 855 to 857; but it is greater than that, in the ratio of 20 to 17 nearly, arising in a great measure from crushing of the plates and the ends of the pillars, through the weight.

Table II. (Table I. continued).—Experiments to obtain the Strength of Solid Uniform Pillars of Low Moor Iron, No. 2, and of Blaenavon Iron, No. 3, the pillars being 10 feet long and  $2\frac{1}{2}$  inches diameter nearly, with the ends turned flat and perpendicular to the axis.

No. of Experiment.	Description of Pillar, 10 feet long, and as above.	Defle <b>ct</b> ion.	Weight producing the deflection.	Breaking weight, or weight with which the pillar sank.	Value of $x$ from formula $w = \frac{W}{D^{3-55}}$ , where $W$ is the breaking weight and $D$ the diameter, the length of the pillar being 10 feet.	Value of $x$ from formula $x = \frac{W}{D^{3.5}}$ .	Remarks.
11.	Solid uniform cylinder of Low Moor Iron, No. 2.  Diameter from eighteen measurements	inch. ·10 ·15 ·20 ·25 ·29 ·33 ·38 ·43 ·51 ·61 ·78 1·23	lbs. 20693 25397 34805 37157 39509 41861 44213 46565 48917 51269 53621 55973	559 <b>73</b> = <b>24</b> ·99 tons.	1bs. 2008:9	2105·2	
12.	Solid uniform cylinder of Low Moor Iron, No. 2.  Diameter from eighteen measurements	·10 ·13 ·16 ·19 ·22 ·25 ·28 ·35 ·39 ·45 ·55 ·67 ·79*	20693 25397 30101 34805 37157 39509 41861 44213 46565 48917 51269 53621 55973	55973† =24·99 tons.	2048-5	2146·1	* See note to Experiment 2, which applies to the cases below marked with an asterisk.  † After bearing the weight 55973 lbs. for half a minute or more, it sank.
13.	Solid uniform cylinder of Blaenavon Iron, No. 3. Diameter from eighteen measurements	·10 ·13 ·16 ·20 ·25 ·34 ·42 ·55 ·66 ·90 1·00*	20693 25397 30101 34805 39509 44213 48917 51269 53621 55973 56914	56914 =25·41 tons	2034-1	2131-8	
14.	Solid uniform cylinder of Blaenavon Iron, No. 3. Diameter from eighteen measurements	·07 ·11 ·14 ·17 ·21 ·28 ·37 ·43 ·54 ·74 ·90*	20693 25397 30101 34805 39509 44213 48917 51269 53621 55973 57855	57855 = 24.83 tons	2096-7	2197.0	

Table III.—Experiments to determine the Strength of Solid Uniform Pillars, 10 feet long and  $2\frac{1}{2}$  inches diameter nearly, from various parts of the kingdom. The pillars were all cast erect and in dry sand, from the same iron model, nearly 12 feet long; they were afterwards cut to 10 feet long, and their ends turned flat and perpendicular to the axis. Two pillars were cast from each kind of iron. The irons are arranged in the order of their strength.

n of	Description of Pillar; each being cast solid, 2½ inches diameter nearly, and turned, 10 feet long, and flat at the ends.	Weights laid on.	Decrement of length.	Deflection of middle.	Breaking weight, or that with which the pillar sank	where W is the breaking weight, D	Value of $x$ from formula $x = \frac{W}{D^{3 \cdot 5}}$ .	Remarks.
1	Old Park Iron, No. 1. Stourbridge. Cold blast. Mean diameter from nine measurements $2.503$ Weight of pillar $151\frac{1}{2}$ lbs.	1bs. 20693 25397 30101 34805 39509 44213 48917 53621 58325 63029 66792	inch065 -076 -087 -097 -106 -116 -127 -138 -149 -161 -171*	inch. ·04  ·06 ·08 ·10 ·18 ·33 ·45*	lbs.	lbs. <b>2571</b> :5	lbs. 2692·2	
2.	Old Park Iron, No. 1. Stourbridge. Cold blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325 63029 65381	·072 ·084 ·095 ·105 ·116 ·127 ·139 ·150 ·161 ·174 ·181*	·07 ·14 ·27 ·31*	65381	2436·8	2552:3	,
3.	Derwent Iron, No. 1. Durham. Hot Blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 57855	·066 ·077 ·089 ·100 ·112 ·123 ·137 ·151 ·164*	·08 ·10 ·16 ·20 ·27 ·43 ·58*	57855	2147-2	2249·2	
4.	Derwent Iron, No. 1. Durham. Hot blast.  Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325 60677 63029 65381 66733	·060 ·071 ·081 ·091 ·102 ·113 ·124 ·135 ·147 ·153 ·159 ·166 ·180	·02 ·06 ·13 ·19 ·23 ·35 ·69	66733	2556-6	2677:4	
5.	Portland Iron, No. 1. Tovine, Scotland. Hot blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 55973 58325 59736	068 079 090 101 112 124 136 150	·09 ·11 ·15 ·19 ·24 ·37 ·46 ·68 to ·73 ·84*	59736	2332·8	2441·8	

<sup>\*</sup> The quantities marked with an asterisk were computed in each case from the two preceding results, as before.

## Table III. (continued).

								-
No. of Ex- periment.	Description of Pillar; each being cast solid, 2½ inches diameter nearly, and turned, 10 feet long, and flat at the ends.	Weights laid on.	Decrement of length.	Deflection of middle.	which the	Value of $x$ from formula $x = \frac{W}{D^{3.65}}$ , where $W$ is the breaking weight, $D$ the diameter, and $x$ the strength of a pillar 1 inch diameter and 10 ft. long.	Value of $x$ from formula $x = \frac{W}{D^{3 \cdot 5}}$	Remarks.
	Portland Iron, No. 1. Tovine, Scotland. Hot blast.  Mean diameter from eight $2.527$ measurements	1bs. 20693 25397 30101 34805 39509 44213 48917 53621 58325 60677 62559	inch065 -075 -086 -097 -107 -118 -130 -145 -159 -170 -179*	inch09 -11 -17 -27 -41 -58 -77 -92*	lbs.	lbs. 2328·3	lbs.	
7.	Calder Iron, No. 1. Lanarkshire. Hot blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325 60677	·069 ·081 ·093 ·105 ·116 ·128 ·140 ·153 ·169 ·177*	·04 ·05 ·08 ·15 ·25 ·38 ·45*	60677	2379·7	2490·7	
	Calder Iron, No. 1. Lanarkshire. Hot blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325 60677	·069 ·081 ·092 ·104 ·115 ·127 ·140 ·154 ·179 ·192*	·06 ·08 ·14 ·28 ·43 ·83 1·13	60677	2283-8	2391.8	
	London Mixture.  One-half old plate iron and one-half Calder iron.  Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 54562	069 082 095 107 121 134 149 183 190*	·04 ·06 ·14 ·20 ·35 ·94 1·06*	54562	2013:7	2109.4	
10.	London Mixture.  One-half old plate iron, and one-half Calder iron.  Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325 59266	·067 ·079 ·091 ·103 ·114 ·126 ·139 ·153 { ·180 to ·182 ·188*	·09 ·12 ·20 ·23 ·33 ·48 ·91 to 1·00 in 2 minutes. 1·10*	59266	2141-9	2244.4	
11.	Level Iron, No. 1. Staffordshire. Hot blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 54091	·070 ·082 ·094 ·107 ·122 ·137 ·155 ·195 ·199*	·08 ·13 ·23 ·34 ·53 1·08 1·14*	54091	2038:8	2135-2	
12.	Level Iron, No. 1. Staffordshire. Hot blast. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 56443	·069 ·082 ·094 ·107 ·121 ·132 ·147 ·167 ·179*	·08 ·13 ·24 ·31 ·45 ·74 ·91*	56443	2179-2	2281-5	

### TABLE III. (continued).

No. of Ex- periment.	Description of Pillar; each being cast solid, 2½ inches diameter nearly, and turned, 10 ft. long, and flat at the ends.	Weights laid on.	Decrement of length.	Deflection of middle.	Breaking weight, or that with which the pillar sank.	Value of $x$ from formula $x = \frac{W}{D^{3.55}}$ , where $W$ is the breaking weight, $D$ the diameter, and $x$ the strength of a pillar 1 inch diameter and 10 ft. long.	Value of $x$ from formula $x = \frac{W}{D^{3 \cdot 5}}$	Remarks.
	Coltness Iron, No. 1. Edinburgh, Hot blast. Mean diameter from eight measurements	lbs. 20693 25397 30101 34805	inch. •065 •077 •089 •100	inch. •05	lbs.	lbs.	lbs.	
	Weight of pillar 151‡ lbs.	39509 44213 48917 53621 56443	·112 ·125 ·139 ·154 ·163*	·10 ·15 ·23 ·44 ·57*	56443	2133-5	2234·2	
14.	Coltness Iron, No. 1.	20693 25397	·068 ·081	•04		. — — — — — — — — — — — — — — — — — — —		
	Edinburgh. Hot blast.  Mean diameter from eight measurements $2.496$	30101 34805	·094 ·108	•06				
	Weight of pillar 149 lbs. 10 oz.	39509 44213 48917	·123 ·139 ·155*	·16 ·30 ·44*	48917	1902-1	1991-1	
	Carron Iron, No. 1. County of Stirling. Hot blast.	20693 25397	·070 ·083	.03				
	Mean diameter from eight measurements	30101 34805	·097	•04				
	Weight of pillar 1493 lbs.	$39509 \\ 44213$	·124 ·137	·06 ·12				
		$48917 \\ 52210$	·154 ·166	·27 •38*	52210	2001.6	2095.7	
16.	Carron Iron, No. 1. County of Stirling. Hot blast.	20693 25397	·076 ·091	•04		The restriction of the state of		
	Mean diameter from eight measurements	30101 34805	·104 ·116	.06				
	Weight of pillar 149 <sup>1</sup> / <sub>4</sub> lbs.	39509 44213	·130 ·144	11				
		48917 53151	·159 ·173*	·32 ·45*	53151	2026·1	2121.6	
	Blaenavon Iron, No. 1. South Wales. Cold blast. Mean diameter from eight measurements	20693 25397 30101	·070 ·082 ·096	·08 ·10				
	Weight of pillar 150 lbs. 14 oz.	34805 39509 44213	·109 ·122 ·137	-21	40027	1000 5	2016.5	
	This was Town No. 1	48917	169	-03	48917	1926.7	2016.5	
18.	Blaenavon Iron, No. 1. South Wales. Cold blast.	20693 25397 30101	·064 ·076 ·088	·05 ·07				
	Mean diameter from eight measurements $\dots$ $2\cdot499$ Weight of pillar $151\frac{3}{4}$ lbs.	34805 39509	·100	•16				
	Weight of pinal 2024 103.	44213 48917	·130 ·154	.60	-			
		49858	·159*	•65*	49858	1930.5	2020-9	
19.	Old Hill Iron, No. 1. Staffordshire. Cold blast.	20693 25397	·088 ·106	.07				
	Mean diameter from eight measurements	30101 34805	·123 ·140	.09				
	Weight of pillar 1513 lbs.	39509 44213	·159 ·188	·13 ·61				
		44683	·191*	·66*	44683	1679.5	1759-0	
20.	Old Hill Iron, No. 1. Staffordshire. Cold blast.	20693 25397	·079 ·094	.04				
	Mean diameter from eight 2.523	30101 34805	·111 ·126	.08				
	Weight of pillar 151 lbs. 10 oz.	39509 44213	·144 ·175	·30 ·56				
		45154	173	·61*	45154	1690.0	1770.0	

# Table III. (continued).

# H	Description of Pillar; each being cast solid, 2\(\frac{1}{2}\) inches diameter nearly, and turned, 10 feet long, and flat at the ends.	Weights laid on.	Decrement of length.	Deflection of middle.	Breaking weight, or that with which the pillar sank.	Value of $x$ from formula $x = \frac{W}{D^{3\cdot55}}$ , where $W$ is the breaking weight, $D$ the diameter, and $x$ the strength of a pillar 1 inch diameter and 10 ft. long.	Value of $x$ from formula $x = \frac{W}{D^{3 \cdot 5}}.$	Remarks.
21.	Second London Mixture. Two-thirds of old metal and one-third of No. 1 best Scotch pig-iron. Mean diameter from eight measurements	lbs. 20693 25397 30101 34805 39509 44213 48917 53621 58325 63029 63499	inch061 -071 -081 -092 -103 -113 -128 -141 -156 -186 -189*	·05 ·08 ·16 ·29 ·46	lbs.	1bs.	1bs	
22.	Second London Mixture. Two-thirds of old metal and one-third of No. 1 best Scotch pig-iron. Mean diameter from eight measurements	20693 25397 30101 34805 39509 44213 48917 53621 58325	·060 ·071 ·081 ·091 ·102 ·114 ·127 ·143 ·159*	·05 ·10 ·18 ·30 ·53 ·70*	58325	2268-0	2374·1	
	Second London Mixture. Two-thirds of old metal and one-third of No. 1 best Scotch pig-iron. Mean diameter from eight measurements	7284 9521 10980		·02 ·13 ·70	11204			
	Second London Mixture. Two-thirds of old metal and one-third of No. 1 best Scotch pig-iron. Mean diameter from eight measurements	7284 9524 9972 10420 10532		·12 ·22 ·30 ·42 ·48	10868			

Table IV.—Solid Pillars cut out of the 10 feet pillars,  $2\frac{1}{2}$  inches diameter, previously experimented upon, and now made 7 feet 6 inches long, quite straight, and with their ends turned flat and perpendicular to the axis, the iron being obtained from various parts of the United Kingdom (Tables III. and II.).

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No. of Ex- periment.	Description of Pillar, 7 feet 6 inches long, and as above.	Weights laid on.	Decre- ment.	Deflec- tion.	Breaking weight, or that with which the pillar sank.	Remarks.
	Old Park Iron, No. 1. Stourbridge. Cold blast. Cut out of the pillar in Table III. Exp. 2. Diameter 2·526 inches.	1bs. 20693 30101 39509 48917 58325 67733 77141 86549 91253 95957 100191	inch058 .078 .098 .118 .138 .157 .176 .196 .207 .221 .234*	·10 ·17 ·25 ·32 ·44 ·55*	100191 lbs. = 44·73 tons.	Broke into four pieces.
2.	Old Park Iron, No. 1. From Exp. 1, Table III. Diameter 2:503 inches.	20693 30101 39509 48917 58325 67733 77141 86549 91253 95487	·059 ·078 ·096 ·113 ·132 ·151 ·171 ·195 ·209 ·222*	·02  ·05 ·08 ·12 ·15 ·23 ·36 ·47 ·57*	95487 lbs. =42·63 tons.	The low result of the breaking weight in this experiment renders it probable that the pillar had sustained some injury by straightening after the former experiment.  This pillar broke in three pieces of nearly equal length, splitting at one end. See Plate XXXIII.
3.	Derwent Iron, No. 1. Durham. Hot blast. From Exp. 4, Table III. Diameter 2.517 inches.	20693 30101 39509 48917 58325 67733 77141 86549 95957 100661 105365	059 076 094 1112 129 146 165 182 205 215 225*	·02 ·04 ·06 ·10 ·15 ·29 ·40 ·51*	$105365  \mathrm{lbs.} = 47.04  \mathrm{tons.}$	
4.	Portland Iron, No. 1. Tovine, Scotland. Hot blast. From Exp 6 Table III. Diameter 2.527 inches.	20693 30101 39509 48917 58325 67733 77141 86549 95957 100661 102543	·059 ·077 ·095 ·112 ·131 ·149 ·168 ·188 ·210 ·235 ·245*	.06 .08 .10 .14 .19 .33 .45 to .59 .69*	102543 lbs. = 45.78 tons	•
5.	Calder Iron, No. 1. Lanarkshire. Hot blast. From Exp. 7, Table III. Diameter 2·490 inches.	20693 30101 39509 48917 58325 67733 72437 77141 81845 86549 91253 93135	·063 ·082 ·102 ·121 ·143 ·164 ·175 ·187 ·200 ·213	.05 .08 .13 .17 .22 .31 .40 .60	93135 lbs. = 41.58 tons.	
6.	London Mixture. One-half old plate iron and one-half Calder Iron. From Exp. 10, Table III. Diameter 2.548 inches.	20693 30101 39509 48917 58325 67733 77141 81845 86549 91253 93605	·057 ·077 ·097 ·116 ·137 ·157 ·179 ·191 ·205 ·224 ·234*	02 06 08 10 16 24 29 41 58 67*	93605 lbs. = 41·79 tons.	This pillar broke in three pieces of nearly equal length, splitting at one end.  It would appear that in this pillar, and in that of Exp. 2, there were more points of contrary flexure than in others. Plate XXXIII.

## Table IV. (continued).

No. of Ex- periment.	Description of Pillar, 7 feet 6 inches long, and as above.	Weights laid on.	Decre- ment.	Deflection.	Breaking weight, or that with which the pillar sank.	Remarks.
7.	Level Iron, No. 1. Staffordshire. Hot blast. From Experiment 12, Table III. Diameter 2:501 inches.	1bs. 20693 30101 39509 48917 58325 67733 77141 81845 86549 91253 95957 98309	inch. ·061 ·082 ·102 ·124 ·165 ·185 ·196 ·207 ·218 ·235 ·244*	.08 .17 .22 .32 .54 .65*	98309  lbs. = 43.89  tons.	
8.	Coltness Iron, No. 1. Edinburgh. Hot blast. From Experiment 13, Table III. Diameter 2.516 inches.	20693 30101 39509 48917 58325 67733 72437 77141 79493 81845 86549 87019	·068 ·088 ·110 ·132 ·153 ·177 ·189 ·201 ·209 ·218 ·251 ·254*	.08 .10 .14 .23 .29 .35 .48 .72 to .81 .92*	87019 lbs. = 38·85 tons.	
9.	Carron Iron, No. 1. County of Stirling. Hot blast. From Experiment 15, Table III. Diameter 2:506 inches.	20693 30101 39509 48917 58325 67733 70085 72437	·078 ·101 ·124 ·149 ·175 ·209 ·222 ·235*	·10 ·12 ·16 ·22 ·32 ·56 ·66 ·76*	72437  lbs. = 32.34  tons.	Comparing the result from this pillar with that from the preceding one on Coltness Iron, the strengths of which should have been equal, as shown by the 10-feet pillars of these irons, I conceive this experiment to be in some degree defective, the pillar having probably been injured by the former experiment upon it.
10.	Blaenavon Iron, No. 1. South Wales. Cold blast. From Experiment 17 or 18, Table III. Diameter 2:5 inches nearly.	20693 30101 39509 48917 58325 67733 71967	·083 ·106 ·128 ·152 ·176 ·208 ·222*	·07 ·11 ·13 ·17 ·27 ·49 ·59*	$71967  \mathrm{lbs.} = 32 \cdot 13  \mathrm{tous.}$	
11.	Low Moor Iron, No. 2. Cut from the pillar in Experiment 12, Table II. Diameter 2.539 inches.	20693 48917 67733 72437 77141 81845 86549 89371		·04 ·07 ·14 ·16 ·21 ·28 ·50 ·63*	89371 lbs. = 39·90 tons.	
12.	Blaenavon Iron, No. 3. Cut out of the pillar in Experiment 13, Table II. Diameter 2.556 inches.	48917 53621 58325 63029 67733 72437 77141 81845 85608		·08 ·10 ·13 ·17 ·22 ·27 ·36 ·49 ·59*	$85608  \mathrm{lbs.} = 38.22  \mathrm{tons.}$	
13.	Low Moor Iron, No. 2. This pillar differs from the preceding ones in being hollow; it was cut out of the pillar in Experiment 2, Table I. Length 7 feet 6 inches. External diameter 2.520 inches. Internal diameter 1.903 inch. Ends flat.	20693 25397 30101 34805 39509 44213 48917 51269 53621 55973		09 13 18 21 26 29 36 40 55 70*	55973 lbs. =24·99 tons.	The strength was probably reduced by being experimented upon before, and afterwards rendered straight; the breaking weight would otherwise have been 63871 lbs. nearly, according to the inverse 1.7th power of the length. Ratio of the thicknesses of the sides '20 to '35, the thin side being compressed.——For form of fracture see Plate XXXIII.

Table V.—Solid Pillars cut out of the 10-feet pillars,  $2\frac{1}{2}$  inches diameter, previously experimented on, and now made 6 feet 3 inches long, quite straight, and with their ends turned flat and perpendicular to the axis, the irons being obtained from various parts of the United Kingdom (Table III.).

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No. of Experiment.	Description of Pillar, 6 feet 3 inches long, and as above.	Weights laid on.	Decrement.	Deflection.	Breaking weight, or that with which the pillar sank.	Remarks.
1.	Derwent Iron, No. 1. Durham. Hot blast. From Exp. 3, Table III.	lbs. 20693 30101 39509	inch. ·024 ·043 ·062	inch.	·	The pillar broke where it had been beaten with a hammer to straighten it after the former experiment, by which its
	Diameter 2.529 inches.	48917 · 58325 67733	·082 ·100 ·119	.02		strength was probably reduced. The situation of the neutral line in this and the following experiment is strongly
	. *	77141 86549 95957	·137 ·158 ·177	·10		marked. The pillar broke at the mid- dle, and was cracked at each end. See Plate XXXIII.
		105365 112438	·202 ·221*	·32 	112438 lbs, = 50·20 tons.	
2.	London Mixture. One-half old plate iron, and one-half Calder iron.	33881 43289 52697 62105	·085 ·105 ·124 ·142	·10		The pillar cracked at both ends, and broke in the middle.  For form of fracture see Plate XXXIII.
	From Exp. 9, Table III. Diameter 2.533 inches.	71513 80921 90329	·161 ·181 ·200	·12		
		99737 104441 109145	·218 ·232 ·247	·18 ·27 ·32		
		113849 114320	·274 ·277*		114320 lbs. = 51.04 tons.	
3.	Level Iron, No. 1. Staffordshire. Hot blast. From Exp. 11, Table III. Diameter 2.518 inches.	33881 43289 52697 62105 71513 80921 90329 99737 109145	·085 ·103 ·123 ·142 ·161 ·179 ·186 ·213 ·232			It broke into a number of pieces. See figure, Plate as above.
		118553 129137 131275	·252 ·269 ·272*	••••	131275 lbs. = 58·60 tons.	
4.	Old Hill Iron, No. 1. Staffordshire. Cold blast. From Exp. 20, Table III. Diameter 2·523 inches.	20693 33881 43289 52697 62105	·065 ·103 ·122 ·146 ·164	.03		The pillar broke across in the middle, and cracked at both ends. See form of fracture, Plate as above.
		71513 80921 85625 90329 93622	·188 ·214 ·230 ·250 ·264*	·22 ·28 ·41	93622 lbs. = 41.79 tons.	
5.	Second London Mixture. About two-thirds of old metal, and one-third of best Scotch pig-iron.	33881 43289 52697 62105	·068 ·086 ·102 ·117			Broke by splitting in the middle and at the ends, one being broken off. See Plate as above.
	From Exp. 21, Table III. Diameter 2.511 inches.	71513 80921 90329 99737	·133 ·149 ·164 ·181	.04		
		109145 118553 123601 127432	·197 ·218 ·233 ·244*	·04 ·14 ·26	127432 lbs. = 56·89 tons,	
		12/402	244	<u> </u>	147 100 50 05 0018.	1 .

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Table V. (continued).

No. of Ex- periment.	Description of Pillar, 6 feet 3 inches long, and as above.	Weights laid on.	Decrement.	Deflection.	Breaking weight, or that with which the pillar sank.	Remarks.
6.	Second London Mixture. About two-thirds of old metal, and one-third of best Scotch pig-iron. From Exp. 22, Table III. Diameter 2-496 inches.	1bs. 33881 43289 52697 62105 71513 80921 90329 99737 109145 116672	inch060 -077 -094 -110 -126 -143 -160 -177 -197 -213*	inch.	116672 lbs. = 52·09 tons.	Broke in the middle and at the ends. See Plate XXXIII.
7.	Second London Mixture. Mean diameter from eight measurements 1.530 in.	7358 9710 12062 14414 16766 19118 21470 23822 26174 26527	·014 ·024 ·032 ·040 ·048 ·057 ·066 ·075 ·091 ·093*		26527 lbs.	For the form of fracture in this and the two following pillars, see Plate as above.
	Second London Mixture. From Exp. 23, Table III. Diameter 1.530 inch.	7358 9710 12062 14414 16766 19118 21470 23822 24763	·027 ·036 ·044 ·051 ·059 ·068 ·077 ·090 ·095*	·06 ·12 ·28 ·38	24763 lbs.	
	Second London Mixture. From Exp. 24, Table III. Diameter 1:541 inch.	7358 9710 12062 14414 16766 19118 21470 23822 25939	·027 ·034 ·043 ·051 ·060 ·068 ·077 ·088 ·098*	·07	25939 lbs.	

Additions to the preceding experiments in Table V., the pillars being cut from the  $1\frac{1}{2}$  inch bars used in Experiments 7, 8 and 9, their lengths being as below, and their ends turned flat.

No. of Exp.	Description of Pillar.	Breaking weight	Remarks.
10.	Length 3 feet. Diameter 1·528 inch. From Experiment 7.	lbs. 57384	
11.	Length 3 feet. Diameter 1 544 inch. From Experiment 8.	70555	
12.	Length 3 feet. Diameter 1.527 inch. From Experiment 9.	57856	
13.	Length 2 feet 6 inches. Diameter 1.531 inch. From Experiment 8.	85138	These pillars broke in the middle, and broke off at each end.  For form of fracture see figures in Plate XXXIII.
14.	Length 2 feet 6 inches. Diameter 1.530 inch. From Experiment 9.	72907	
15.	Length 2 feet 6 inches. Diameter 1.515 inch. From Experiment 7.	70555	

Table VI.—Solid Pillars cut out of the 10-feet pillars,  $2\frac{1}{2}$  in diameter, from various parts of the kingdom, the pillars being now rendered straight and cut to 5 feet long nearly, and of which the first three have their ends turned flat and perpendicular to the axis, and the others have their ends rounded, so that the force is applied along the axis of the pillar.

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No. of Ex- periment.	De	escription of Pillar, 5 feet ong nearly, and as above.	Weights laid on.	Decrement of length.	Deflection of middle.	Breaking weight, or that with which the pillar sank.	Remarks.
1.	Stat Fro	Hill Iron, No. 1. ffordshire. Cold blast. m the pillar in Exp. 19, lable III. gth 59.9 inches.	lbs. 20693 38644 48052 57460 66868 76276 85684 95092 104500 113908 117612 123212 128812	inch038 -074 -090 -109 -128 -146 -165 -184 -202 -223 -234 -247 -264	·09 ·16 ·20 ·27	1950971	Supposing that the strength of pillars of the same diameter varies inversely as the 17th power of the length, the breaking weight of this, if 5 feet long, would be 135592 lbs., and of the following pillar 122863 lbs.; mean 129227 lbs. = 57.69 tons.  For form of fracture see figure in Plate XXXIII.
			135977	-285*	·50*	$\frac{135977 \text{ lbs.} = 60.70 \text{ tons.}}{$	
2.	From programmer T	Hill Iron, No. 1. m the same pillar as the receding experiment, able III., Exp. 19. gth 59.9 inches.	38644 48052 57460 66868 76276 85684 95092 104500 113908 117612 123212	·097 ·118 ·137 ·157 ·178 ·198 ·220 ·243 ·270 ·285 ·308*	·05 ·07 ·10 ·13 ·17 ·21 ·34 ·46 ·64*	123212 lbs. = 55·01 tons.	One of these two pillars broke across in the middle, and near to each end.
3.	From	enavon Iron, No. 1. m the pillar in Exp. 17 r 18, Table III. gth 5 feet.	20693 38644 48052 57460 66868 76276 85684 95092 104500 113908 117612	·052 ·080 ·096 ·111 ·127 ·145 ·161 ·179 ·202 ·239 ·253*	•••••	117612 lbs. = 52·51 tons.	
4.		Low Moor Iron, No. 2. From the pillar in Exp. 11, Table II. Length 59.5 inches. Diameter 2.553 ins.	48917 53621 58325 63029 65381		·04 ·07 ·12 ·26 ·60	<u> </u>	5'v 0"
	Pillars with their ends rounded (see figure)					65381 lbs., or 64458 lbs. for a length of 5 feet.	The ends of the pillar were crushed by the pressure, the diameter of the crushed part being '73 inch, and the crushing had been perceptible, and apparently one-third to half an inch when two-thirds of the breaking weight was laid on. There were several cracks at the ends of the pillar through the pressure. The strength had probably been much increased by the flattening of the ends. See Plate XXXIII. for the forms of fracture in this and the two following pillars.
5.	Pillars v	Blaenavon Iron, No. 3. From the pillar in Exp. 14, Table II. Length 59.5 inches. Diameter 2.546 ins.	53621 58325 63029 65381 67733 70085		·05 ·07 ·10 ·13 ·18 ·26	71967 lbs., or 70950 lbs. if the length had been 5 feet.	With a pressure of three-fourths of the breaking weight, the diameter of the compressed circle at the ends was about 6 inch, and the diameter after fracture was 72 inch nearly.

No. of Ex- periment.	Description of Pillar, 5 feet long nearly, and as above.	Weights laid on.	Decrement of length.		Breaking weight, or that with which the pillar sank.	Remarks.
6.	Blaenavon Iron, No. 3. From the pillar in Exp. 14, Table II., same as last. Length 59.5 inches. Diameter 2.546 inches.	lbs. 58325 67733 72437	inch.	·14 ·22 ·30	73378 lbs., or 72342 lbs.	Diameter of crushed end of pillar 63 with weight 30101 lbs., and after fracture 79.  Many small cracks in the end.

The strength of the three last pillars had probably been much increased by the flattening at the ends. If the ends of the pillar had been small and not crushed, the strength would have been about one-third of that of a pillar of the same diameter and length, with its ends turned flat and well-bedded.

The crushing at the ends was not only shown in the pillar, but in the hard steel plates against which it was pressed. The case is analogous to that of a wheel sinking into a hard surface upon which it should roll.

Table VII.—Transverse strength of the 10-feet Pillars, cast  $2\frac{1}{2}$  inches diameter, and now cut to 5 feet long, laid on supports 4 feet 6 inches asunder, and broken by weights laid on in the middle.

Portla	and Iron,	No. 1. Diam	eter <b>2·493</b>	inches. Distance between the supports 4 feet 6 inches.		
First half of bar. Second half of bar.		f of bar.				
Weights laid on.  lbs. 560 1120 1680 2240 2800	inch. ·06 ·145 ·22 ·31 ·40	Weights laid on.	inch. •063 •113 •215 •31 •40	Breaking weight of first bar		
3360 3920 4324 broke.	.51 .66 .75* ult. defl.	3360 3920 4480 4940 broke.	·50 ·63 ·77 ·87* ult. defl.	Breaking weight for 7-ft. 6-in. bar = 2779		
Calc	ler Iron, N	No. 1. Diame	eter 2.519	inches. Distance between the supports 4 feet 6 inches.		
560 1120 1680 2240 2800 3360 3640 3920 4324 broke.	05 14 22 30 38 49 55 59 64* ult. defl.	560 1120 1680 2240 2800 3360 3484 broke.	·06 ·14 ·21 ·30 ·395 ·49 ·51* ult. defl.	Breaking weight of first bar		
1120 1680 2240 2800 3360 4044 broke.	18 295 41 56 75 96* ult. defl.	No. 1. Diam  560 1120 1680 2240 2800 3360 3652	.09 .19 .31 .43 .59 .77 .84*	Breaking weight of first bar		

TABLE	VII.	(continue	d).
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First half	of bar.	Second hal	f of bar.	
lbs. 560 1120 1680 2240 2800 3080 3428 broke.	inch07 .155 .25 .35 .46 .54 .62* ult. defl.	Weights laid on.   1bs.   560   1120   1680   2240   2800   3080   3428   broke.	inch08 -16 -26 -37 -48 -55 -63* ult. defl.	Breaking weight of first bar3428 Breaking weight of second bar3428 Mean 3428 lbs.; whence Breaking weight for 10-ft. bar $= 1543$ Breaking weight for 7-ft. 6-in. bar $= 2057$

Table VIII.—Crushing weights of short cylinders of different kinds of Cast Iron, cut from the bars,  $2\frac{1}{2}$  inches diameter previously used, and now turned to be  $\frac{3}{4}$  inch diameter nearly, and  $1\frac{1}{2}$  inch high. The results are means from three or four experiments on each kind of iron. The specimens were usually cut out of the iron between the centre and the circumference of the bar, denominated the medium part. In several cases they were cut out of the centre of the bar, and sometimes out of the circumference.

Description of Iron.	Diameter of specimen.	Crushing weight of specimen.	Crushing weight per square inch of section.
Old Park Iron, No. 1. Medium.	inch. •747 •747	lbs. 38333 38862	$\begin{bmatrix} \text{lbs.} \\ 87466 \\ 88674 \end{bmatrix} 88070 \text{ lbs.} = 39.32 \text{ tons.}$
Old Park Iron, No. 1. Centre.	747 747 747 747	32923 32453 32923 32571	$ \begin{array}{c} 75122 \\ 74050 \\ 75122 \\ 74319 \end{array} \} 74653 \text{ lbs.} = 33.33 \text{ tons.} $
Derwent Iron, No. 1. Medium.	·747 ·747 ·747 ·747	43096 42684 42684 41861	$ \begin{vmatrix} 98334 \\ 97394 \\ 97394 \\ 95517 \end{vmatrix} 97160 \text{ lbs.} = 43.37 \text{ tons.} $
Coltness Iron, No. 1. Medium.	·747 ·747 ·747 ·747	27749 27925 27455 27396	$     \begin{cases}       63316 \\       63718 \\       62646 \\       62511     \end{cases}     \begin{cases}       63048 \text{ lbs.} = 28 \cdot 14 \text{ tons.} \\       62646 \\       62511     \end{cases} $
Blaenavon Iron, No. 1. Medium.	·748 ·748 ·748	31336 31395 30748	$   \begin{array}{c c}     71310 \\     71444 \\     69972   \end{array}   $ $   \begin{array}{c c}     70909 \text{ lbs.} = 31.66 \text{ tons.} \\   \end{array} $
Level Iron, No. 1. Medium.	·749 ·748 ·750 ·749	30277 30101 30101 29748	68716 68500 68135 67516 68217 lbs. = 30 45 tons.
Carron Iron, No. 1. Medium.	·750 ·750 ·751 ·751	31747 29278 30101 30101	71860 66272 67953 67953 67953 68509 lbs. = 30.58 tons.
London Mixture. Medium.	·749 ·750 ·749	36157 35452 35452	82061 80247 80461 80923 lbs. = 36.08 tons.
Calder Iron, No. 1. Medium.	·750 ·749 ·750 ·749	38039 35040 38980 37333	86103 79526 88233 84730 84648 lbs. = 37.79 tons.

Table VIII. (continued).

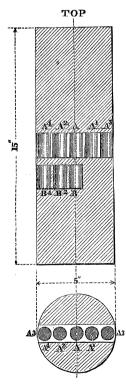
Description of Iron.	Diameter of specimen.	Crushing weight of specimen.	Crushing weight per square inch of section.
Portland Iron, No. 1. Medium.	inch747 -748 -750 -750	lbs. 41567 41273 42743 41391	$\begin{bmatrix} 1\text{bs.} \\ 94846 \\ 93923 \\ 96750 \\ 93690 \end{bmatrix} 94802 \text{ lbs.} = 42\cdot32 \text{ tons.}$
Old Hill Iron, No. 1. Medium.	·749 ·747 ·750	24255 24221 23843	$\begin{bmatrix} 55049 \\ 55266 \\ 53969 \end{bmatrix} 54761 \text{ lbs.} = 24.45 \text{ tons.}$
Low Moor Iron, No. 2. Medium.	·748 ·748 ·747	34570 33511 33982	78669 76260 77539 77489 lbs. = 34·59 tons.
Low Moor Iron, No. 2. Centre.	·742 ·741 ·742	29631 27749 28690	$ \begin{array}{c} 68525 \\ 64346 \\ 66349 \end{array} \right\} 66407 \text{ lbs.} = 29.65 \text{ tons.} $
Blaenavon Iron, No. 3. Medium.	•737 •737 •737	35511 35746 35629	$   \begin{array}{c}     83241 \\     83792 \\     83517   \end{array}   \left. \begin{array}{c}     83517 \text{ lbs.} = 37.28 \text{ tons.} \\   \end{array}   \right. $
Blaenavon Iron, No. 3. Centre.	·747 ·747 ·747	38568 34805 27396	$ \begin{array}{c} 88003 \\ 79416 \\ 62511 \end{array} \right\} 76643 \text{ lbs.} = 34 \cdot 22 \text{ tons.} $
Second London Mixture.  Medium. From $2\frac{1}{2}$ -inch. pillar, as all above have been.	·747 ·747 ·747	41626 41861 41861	$ \begin{array}{c} 94980 \\ 95517 \\ 95517 \end{array} \} 95338 \text{ lbs.} = 42.56 \text{ tons.} $
Second London Mixture. Centre. From $2\frac{1}{2}$ -inch. pillar, as all above have been.	·747 ·747 ·748	34805 34099 34335	$   \begin{array}{c}     79411 \\     77806 \\     78135   \end{array}   \left. \begin{array}{c}     78451 \text{ lbs.} = 35.02 \text{ tons.} \\     \hline     \end{array}   \right. $
Second London Mixture. Medium. From 1½-inch pillar.	·750 ·750 ·750	49858 48917 48447	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$
Second London Mixture. Centre. From 1½-inch pillar.	·750 ·750 ·750	41861 48094 47976	
Low Moor Iron, No. 2. From a hollow pillar 4 inches diameter and ½ inch thick. The height of the first two of these was 72 inch, and of the last 1.502 inch.	·420 ·422 ·421	12415 11944 11592	89610 85395 83273 87502 lbs. = 39.06 tons.
Low Moor Iron, No. 2.  From the thin ring of a hollow pillar about $3\frac{1}{2}$ inches diameter.  Height of specimens '53 inch.	·298 ·300 ·298	8010 8255 8115	114845 116785 116350 115993 lbs. = 51.78 tons.
Low Moor Iron, No. 2.  From the thin ring of a hollow pillar about $3\frac{1}{2}$ inches diameter.  Height of specimens 53 inch.	·297 ·296 ·296	7905 7555 7345	$ \begin{array}{c} 114105 \\ 109790 \\ 106740 \end{array} \} 110212  \text{lbs.} = 49 \cdot 20  \text{tons.} $

Of the different irons tried in the preceding experiments on pillars, whether solid or hollow, the external part of the casting was always harder than that near to the centre, and the iron of the external ring of a hollow casting was very hard, the hardness increasing with the thinness. Thus, in solid pillars  $2\frac{1}{2}$  inches diameter of Low Moor iron, No. 2, the crushing force per square inch of the central part was 29.65 tons, and that of the intermediate part near to the surface was 34.59 tons, whilst the external

ring,  $\frac{1}{2}$  inch thick, of a hollow cylinder 4 inches diameter, of which the outer crust had been removed, was crushed with 39·06 tons per square inch; and external rings of the same iron thinner than half an inch, required from 49·2 to 51·78 tons per square inch to crush them. These facts show the great superiority of hollow pillars over solid ones of the same weight and length.

# Table VIII. (continued).—Casting of larger dimensions than before used.

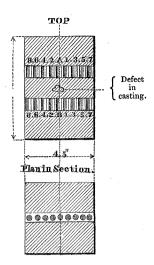
To ascertain whether the internal strength of larger pillars varied in the same manner as that of smaller ones, a cylindrical casting was made 5 inches diameter and 15 inches long. It was cast vertically, from Blaenavon iron. Through the axis of this cylinder, a slab, 15 inches long, 5 inches broad, and about 1 inch thick, was taken. Across the middle of this slab five cylinders,  $1\frac{1}{2}$  inch long and  $\frac{3}{4}$  inch diameter, were obtained at equal distances from each other, the middle one being in the centre, and the outer ones as near as possible to the sides. Three other small cylinders of the same size were also obtained, the positions of which were as in the figure.



Mark on Cylinder.	Diameter of Cylinder.	Crushing weight of Cylinder.	Crushing weight per square inch.
A4 B4 A2 B2 A B A1 A3	inch. •749 •748 •750 •748 •748 •748 •748 •748	1bs. 27749 30101 27749 27279 27749 28690 28690 29160	lbs. 62979

Comparing the results of the experiments above, it appears that the external part of the casting was somewhat stronger than the internal. But the variation was only from 62 to 66, and therefore much less than was obtained from the smaller masses.

To obtain some additional information as to the distribution of the metal in a large casting, one was formed 9 inches square and 12 inches long, or upwards. It was cast erect from Derwent iron, No. 1, and was then placed upon a slotting machine, and cut directly across in the middle. It was found, however, that there was a hole in the centre, about  $\frac{3}{4}$  inch or 1 inch in diameter; transverse slices were therefore cut, one from above and one from below the central defective piece; these two slices, each 9 inches square and 1 inch thick nearly, were divided from one side to the other, passing through the centre of each, into nine equal parts, and from each part cylinders were cut, all of the same diameter,  $\frac{3}{4}$  inch, and length  $1\frac{1}{2}$  inch, four on each side of the central one. See figure annexed.



Upper Slice of Mass.

Mark on Cylinder.	Diameter of Ditto.	Crushing weight of Ditto.
	inch.	lbs.
8	•748	31512
6	•747	32923
4	•748	31042
2	.747	37157
${f A}$	.748	37627
1	•748	39980
3	•748	39039
5	•748	35275
7	•748	31983

Lower Slice of Mass.

Mark on Cylinder.	Diameter of Ditto.	Crushing weight of Ditto.
8	inch. •748	lbs. 30571
6 4	·748 ·749	31512 32924
2 B	•748 •748 •748	30571 29160 30101
3 5	·748 ·749	32924 32924
7	•749	33394

The iron in this mass seems to have been extremely irregular, particularly in the upper part of the casting. In the part marked B, which was cast under the greater head of metal, the iron from three divisions near the centre, or one-third of the distance across, was crushed with somewhat less weight than that from the external part, the greatest difference being from 29160 at the centre, to 33394 at the side. The mean resistance of the three central cylinders in this slice is 29944 lbs., and of the six external ones 32375 lbs., the difference being much less than in the iron of the smaller castings tried.

The iron in the upper part, marked A in the figure, being cast under a less head of metal than the former, and floating at the top of the softer metal below, was considerably stronger than the other. The central portion of this was, contrary to all our previous experience, much harder than the rest, and varied from 31512 and 31983 at the sides, to 37627, 39039 and 39980 near to the middle. To explain this strange anomaly in the absence of other knowledge, it seems not improbable that some additional heavier metal had been poured into the top part of the mould, when that in the lower part had in some degree solidified.

From the experiments upon the two last castings, which are on a larger scale than those which precede them, it appears that the difference of hardness between the external and internal parts of a large casting is much less than in a small one, and may frequently be neglected.

#### APPENDIX.

The following experiments were made after the others had been offered to the Royal Society, or during the months of October and November 1857, and as the former solid pillars were from various parts of the kingdom, and usually of the iron termed No. 1, I was desirous of adding the results from a few solid pillars of the quality No. 2. These latter pillars were mostly cast circular, and were made 10 feet long and  $2\frac{1}{2}$  inches diameter, with their ends turned flat as before. To these were added pillars of nearly the same weight, of a square and a triangular form of section, to ascertain the comparative strength of pillars whose sections were circular, square, and triangular.

The cylindrical pillars, in the experiments of which an abstract is given below, though of different irons, were all cast from the same model; and the square and the triangular pillars were of the same kind of iron as two of the circular ones (the Calder, No. 2), and were intended to be of the same weights as these. All the pillars were made uniform throughout their lengths, 10 feet long each, and turned flat at the ends.

As the weights of the different pillars did not differ widely from 150 lbs. each, the breaking weight of each is reduced to what it would have been if the weight had been 150 lbs.

The	reduced	results	are	ลร	helow

Pillars broken.	Second quality of iron, or that of No. 2.	Assumed weight of Pillars.	Breaking weights and forms of section.
1st pillar	Coltness Iron, No. 2	lbs. 150	lbs. <b>50328</b>
2nd pillar	Old Park Iron, No. 2 Blaenavon Iron, No. 2	$\begin{array}{c} 150 \\ 150 \end{array}$	0.54736 0.55611
	Calder Iron, No. 2	150	55658 Mean.
5th pillar	Calder Iron, second specimen	150	$\begin{array}{ c c c } \hline \\ 54940 \end{array} > 55299  \text{lbs.}$
6th pillar	Calder Iron, No. 2	150	49185 $51537 $ lbs.
	Calder Iron, second specimen		53890
8th pillar	Calder Iron, No. 2	150	57332 $61056$ lbs.
9th pillar	Calder Iron, second specimen	150	64780

From the three last mean results it appears that the strengths of circular, square, and triangular solid pillars of the same quality, weight and length, vary as 55299, 51537, and 61056, the last being the strongest. This last conclusion, respecting the strength of triangular solid pillars, may not be considered void of importance. See Notes to experiments 8 and 9, Table IX. following.

Whence it appears that solid triangular pillars with flat ends are stronger than those with either circular or square sections; but this seems not to be the case in pillars with rounded ends, for from my former experiments\*, a pillar, whose section is +, as in the connecting rod of a steam-engine, the ends being moveable, is very weak to bear a strain

<sup>\*</sup> Philosophical Transactions, 1840, pages 413 and 455.

as a pillar; and indeed less than half the strength of a hollow cylindrical pillar of the same weight and length, rounded at the ends.

Comparing the results of the cylindrical pillars tried, it appears that those of the second quality, or No. 2, are usually weaker than those of No. 1 of the same iron. In the irons below we have the breaking weights of pillars, 10 feet long and  $2\frac{1}{2}$  inches diameter, of the first and second quality, as follow:—

	lbs.			lbs.
Old Park Iron, No. 1 6	66086	No. 2, or second quality .		55032
Calder Iron, No. 1 6	60677	No. 2, or second quality .	•	55032
Coltness Iron, No. 1 5	52680	No. 2, or second quality .		50328

The properties of the irons of the second quality are deduced from the following Table of experiments; those of irons termed No. 1 may be obtained from the second summary of results, page 871.

#### TABLE IX.

Cast-iron pillars of the quality No. 2, supplied by Messrs. Easton and Amos, October and November 1857. The pillars were cast to be  $2\frac{1}{2}$  inches diameter, and each was cut 10 feet long, the ends being turned flat and parallel.

Experiment 1.

Cylindrical Pillar, Coltness Iron, No. 2.

Mean diameter from eight admeasurements 2·494 inches.

Weight of Pillar 150 lbs.

Weights laid or	n. Deflections.	Decrements of length.
lbs.	inch.	inch.
20693	·17	$\cdot 057$
25397	·20	$\cdot 078$
30101	•24	$\cdot 091$
34805	·31	·105
39509	$\cdot 42$	·120
44213	·62	$\cdot 135$
48917	.98	$\cdot 163$
50328	Broke with this in two places near	the middle.

As the weights of the following pillars differed generally but little from that above, and a leading object being to compare the results together, the breaking weight has been reduced, as before mentioned, in each case, to what it would have been for a pillar weighing exactly 150 lbs.

## Experiment 2.

Cylindrical Pillar, Old Park Iron, No. 2. Mean diameter from eight admeasurements 2:503 inches. Weight of Pillar 150 lbs. 13 oz. = 150.812 lbs.

Weights laid on.	Deflections.	Decrements of length.
lbs.	inch.	inch.
20693	April 100 miles	.084
25397	.07	$\cdot 098$
30101	<del></del>	·111
34805	:09	·124
39509	·12	.138
44213	·18	$\cdot 152$
48917	•30	·166
$\boldsymbol{53621}$	.78	·194
55032 broke it.	It broke exactly in the middl	e. —

33032 broke it. It broke exactly in the middle.

lbs. lbs. lbs. lbs.

Whence 150.812:150::55032:54736=reduced breaking weight.

## Experiment 3.

Cylindrical Pillar, Blaenavon Iron, No. 2. Mean diameter from eight admeasurements 2.486 inches. Weight of Pillar 148 lbs. 7 oz.=148.437 lbs.

Weights laid on.	Deflections.	Decrements of length.
lbs.	inch.	inch.
20693	***************************************	.046
25397		$\cdot 059$
30101		$\cdot 073$
34805	$\cdot 02$	.086
39509	$\cdot 09$	.098
44213	·15	·112
48917	•30	·128
<b>5</b> 3621	$\cdot 72$	·156
55032 sank.	Not broken. —	- Charles

lbs. lbs. lbs. lbs.

Whence 148.437:150::55032:55611=reduced breaking weight.

#### Experiment 4.

Cylindrical Pillar, Calder Iron, No. 2.

Mean diameter from eight admeasurements 2.469 inches.

Weight of Pillar 148 lbs. 5 oz. = 148.3125 lbs.

Weights laid on.	Deflections. D	ecrements of length.
lbs.	inch.	inch.
20693	·11	$\cdot 070$
25397	$\cdot 13$	.084
30101	$\cdot 16$	$\cdot 097$
34805	$\cdot 19$	.110
39509	$\cdot 24$	$\cdot 122$
44213	$\cdot 30$	$\cdot 130$
48917	$\cdot 45$	$\cdot 147$
53621	$\cdot 72$	$\cdot 165$
55032 broke it.	Broke $4\frac{1}{2}$ inches from the mide	dle. —

lbs. lbs. lbs. lbs.

Whence 148:312:150::55032:55658=reduced breaking weight.

### Experiment 5.

Cylindrical Pillar, Calder Iron, No. 2.

Mean diameter from six admeasurements 2.497 inches.

### Weight of Pillar 150.25 lbs.

Weights laid	on. Deflections.	Decrements of length.
lbs.	inch.	inch.
20693	17	$\cdot 077$
25397	$\cdot 21$	.089
30101	$\cdot 25$	$\cdot 102$
34805	$\cdot 29$	$\cdot 117$
39509	34	·131
44213	.38	$\cdot 144$
48917	·55	·158
53621	•94	$\cdot 182$
55032	Sank without fracture.	<del>Vysenquis</del> .

lbs. lbs. lbs. lbs.

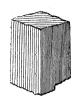
Whence 150·25: 150:: 55032: 54940=reduced breaking weight.

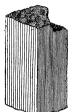
$$\frac{55658 + 54940}{2} = 55299 \text{ lbs.},$$

the mean strength of a cylindrical pillar of Calder Iron, No. 2, 10 feet long and  $2\frac{1}{2}$  inches diameter nearly.

Experiment 6.—Square Pillar, Calder Iron, No. 2. Side of square from eight admeasurements 2·194 inches. Weight of Pillar 146 lbs. 5 oz.=146·312 lbs.

Weights laid o	n.	Deflections.	Decrements of length.
lbs.		inch.	inch.
20693		·19	$\cdot 068$
25397		$\cdot 23$	.081
30101		$\cdot 27$	.094
34805		$\cdot 35$	·108
39509		·47	$\cdot 122$
44213		$\cdot 69$	$\cdot 142$
47976	Broke.	f britains	





Broke about 5 inches from the middle by bending in the diagonal.

lbs. lbs. lbs.

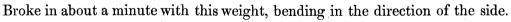
Whence 146.312:150::47976:49185=reduced breaking weight.

The figures standing over each other represent the pillar, at the place of fracture, near the middle; and the small additional piece, that of the part broken out from one corner by compression.

Experiment 7.—Square Pillar, Calder Iron, No. 2. Side of square from eight admeasurements 2·221 inches.

Weight of Pillar 149.25 lbs.

Weights laid on.	Deflections.	Decrements of length.	
lbs.	inch.	inch.	
20693	·14	.077	
25397	$\cdot 17$	∙090	
30101	$\cdot 21$	·103	
34805	$\cdot 27$	·115	Ш
39509	•34	$\cdot 128$	
44213	$\cdot 45$	·143	75.5
48917	.64	·160	
53621	1.14	·195	1



lbs. lbs. lbs.

Whence 149.25: 150:: 53621: 53890=reduced breaking weight.

$$\frac{53890 + 49185}{2} = 51537 \text{ lbs.},$$

the mean strength of a square pillar 150 lbs. in weight, its length being 10 feet.

The vertical figures represent the pillar at the place of fracture, which in this case was different in form from the preceding one, and along the side; these and the smaller figure, representing the transverse section, show the small size of the compressed part.





Experiment 8.—Triangular Pillar, Calder Iron, No. 2. Mean from six admeasurements of sides of triangle near the middle 3.545 inches.

Weight of Pillar 158.75 lbs.

337.1.1 1.11	D-0-4'		
Weights laid	on. Deflections. Decren	nents of length.	644
lbs.	inch.	inch.	الرال ال
20693	$\cdot 23$	.080	4
25397	$\cdot 27$	$\cdot 092$	
30101	•33	$\cdot 107$	
34805	$\cdot 39$	$\cdot 121$	
39509	$\cdot 46$	·134	
44213	•53	$\cdot 147$	
48917	$\cdot 64$	·161	
53621	$\cdot 79$	$\cdot 177$	
<b>58325</b>	1.06	203	
60677	Broke, having bent parallel to one side.	***************************************	

lbs. lbs. lbs. Whence 158.75:150::60677:57332 = reduced breaking weight.

This triangular pillar differed but little, in the form of its fracture, from the preceding rectangular one, as shown by the figure of the compressed part along its side; its strength, however, is greater than that of any other yet tried.

Experiment 9.—Triangular Pillar, Calder Iron, No. 2.

Mean breadth of the sides from nine admeasurements, in three parts, near to the middle 3.47 inches.

Weight of Pillar 155.75 lbs.

Weights laid on	. Deflections. I	Decrements of length.	
lbs.	inch.	inch.	
20693	.08	.070	
25397	$\cdot 09$	.082	
30101	·11	$\cdot 093$	
34805	·14	·105	
39509	·18	·117	
44213	$\cdot 22^{\circ}$	·130	
48917	$\cdot 27$	·141	
53621	•36	·154	
58325	•48	·169	
63029	$\cdot 69$	·185	
67263  I	Broke. See form of fra	acture. —	
		bs.	
Whence 15	5.75:150::67263:64	780 = reduced	
breaking weig	· · · · · · · · · · · · · · · · · · ·		

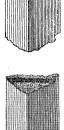
 $\frac{57332 + 64780}{} = 61056 \text{ lbs.},$ 

lbs.

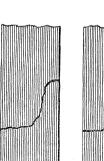
the mean strength of a triangular pillar, 10 feet long, if its weight were 150 lbs.







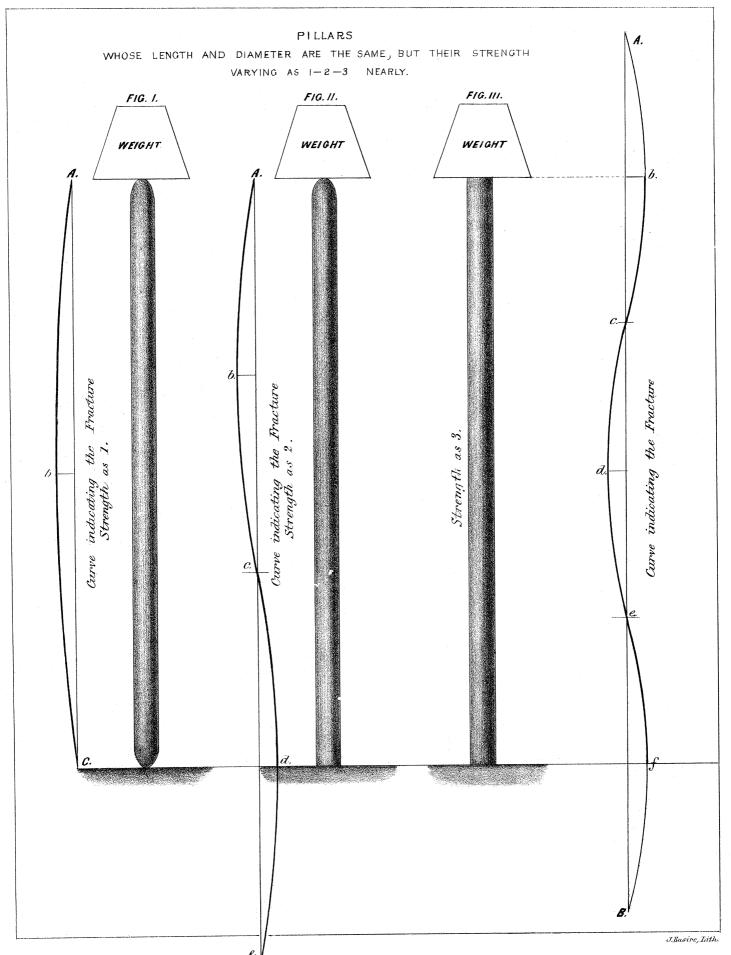




This pillar broke, different from the last, by compression of the edge, showing great additional strength.

Note. The last pillar broke by the compression of one of the edges, and the iron in these, being harder than that in the thicker parts, would offer greater resistance to crushing than in them; and hence the strength of the pillar would be increased (see the experiments in Table VIII.). The part subjected to crushing would in this pillar be at a greater distance from the centre of the section than in the others; and from my numerous experiments on the resistances to crushing and to tearing asunder in cast iron, it appears that the former is, on the average, about six times as great as the latter. See my experiments in the Report of the Commissioners on the Application of Iron to Railway Structures, page 15. From these and other causes the greater strength of a solid triangular pillar of cast iron, above one of a circular or square section, may be inferred; a conclusion not without importance in some applications of the metal.

PILLARS. OF STRENGTH ON THE APPARATUS USED IN MR HODGKINSON'S EXPERIMENTS



<-----H O ILLA S. . . . . . . P R L L TABLE IV. TABLE 1. EXPT\$ 1 & 2. EXPT 13. EXPT4. Νò L o w M O O R 2. LO ₹ 3" x

J. Basire, Lithog.

